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# Viability of Concentrator Photovoltaics for Electricity Production in the United States

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# CHAPTER 1

# Introduction

**1.1. Motivation.** In the United States, our way of life would not be possible without the abundant availability of electricity. It is currently inexpensive, easily accessible, and seemingly endless in nature. This electricity plays a fundamental role in both residential and commercial environments, making it an absolutely crucial element of our economy. In 2015 alone, the US consumed 4,087 TWh of electricity, partially composed of 33.2% produced by coal-fired generators, 32.6% from natural gas, and 19.5% from nuclear reactors.[1] A shortage of electricity could have sweeping negative effects on our country, and thus the entire world. Therefore, it is vital to ensure there is a sufficient supply for the future.

Our current energy needs are satisfied, and their sources are sufficient to meet many more years of consumption at present rates. However, there are flaws in our current system for producing energy. Fossil fuels make up the largest share of our energy portfolio at 65.8%, and there are significant drawbacks to its use. Not only are fossil fuels a finite resource, but we are also burning through them at an alarming pace. There have been calculations that attempt to estimate how long Earth's fossil fuel reserves will last, but they vary as new technologies develop which allow access to previously inaccessible fuel. Proponents of continued fossil fuel use would argue that we will find more reserves, but at our current rate of consumption, it is unlikely that those reserves will last more than a few hundred years. If we do not begin investigating alternatives to fossil fuels, we may face an energy deficit, which would make it difficult to implement new energy systems.

In addition, the collection and combustion of fossil fuels can be damaging to the environment. Burning the organic molecules stored in the Earth releases $CO_2$ gas as a product of the chemical reaction. This has been proven, through an analysis of carbon isotope ratios, to be increasing the concentrations of $CO_2$ in the atmosphere.[2] Due to the interactions of the $CO_2$ molecule with infrared wavelengths of light, a larger share of Earth's radiation is prevented from escaping into space through an increased greenhouse effect. This additional trapped heat can cause several negative effects across the globe, such as ocean acidification, rising sea levels, the melting of the ice caps, and more severe weather events. In this way, the combustion of fossil fuels has a negative impact on our environment.

Next, we can explore environmental damage as a result of the collection of fossil fuels. As an example, there is the practice of hydraulic fracturing, or more commonly known as "fracking." In this procedure, water is mixed with fracturing fluid, and sent deep into the earth to fracture the shale rock layer and release the natural gas and oil trapped within. Typically, less than 30% of this water is subsequently recovered from the well.[3] Since several million gallons of water are used per well, this results in releasing a large volume of chemicals into the environment, potentially contaminating our precious clean drinking water and more. Essentially the only factor in favor of the continued use of fossil fuels is its economic superiority to alternative energy sources. Although fossil fuels succeed economically, the threats of global warming and environmental damage are too great to be ignored.

The other majority of our energy portfolio consists of nuclear energy. Nuclear is also currently unsustainable due to the production of nuclear waste that cannot be safely disposed of long term. The waste products are a permanent liability, posing the constant threat of exposure to humans and the environment. Until an effective method of disposal is discovered, we should work to decrease the production of radioactive waste products in the name of precaution. Also, the rare occurrence of a reactor meltdown is possible in extreme conditions, and should not be ruled out. Both the reactors and the nuclear waste storage sites are also vulnerable to sabotage, making them appealing targets for terrorist groups. At this point, the need for a sustainable source of energy, which can be harvested without significant negative impacts, becomes apparent. Considering that both nuclear and fossil fuels are not optimal candidates for our future energy sources, alternatives must be put in place for up to 85.3% of our electricity in order for our energy systems to be sustainable. It is clear that fossil fuels are not a sustainable energy source. Despite their abundance and usefulness, the consequences of its use are too great and ideally should be avoided. Nuclear is an accident waiting to happen, and its continued use increases the amount of radioactive waste for our country to dispose of. The effort that exists to replace these energy systems is crucial in preserving our planet while continuing to fuel our economy.

History has proven that the success of mankind is based on utilizing an abundant source of energy. The use of hydropower in mills, farm animals to pull plows, and even human slaves to complete tasks played a critical role in increasing the production of goods or construction of buildings that paved the way for the success and expansion of civilization. In the same way, to continue expanding our economy, we must provide ourselves with a sufficient source of energy. Although growth in electricity consumption has plateaued in recent years, it is valid to assume that our consumption will increase in the future as the population increases. As older generating facilities reach the end of their working lives, this means we will need to replace them and build additional power plants. In place of building more coal, nuclear, and natural gas generators, we should take advantage of this opportunity and develop sustainable technologies to provide a secure energy future for our country. Previously, we have largely allowed the free market to determine our energy portfolio. In our situation, sustainability should be the number one priority.

In addition, our country would gain an economic advantage by investing in and developing these technologies. Other regions of high growth, such as China, will have a high demand for energy sources as well. If American companies can find a way to lower the costs of alternative energy sources, we will benefit from the construction of their energy infrastructure. In effect, this would partially offset the initial costs of developing and implementing the sustainable energy systems. In this way, our nation has the power to accelerate a worldwide transition to sustainable energy.

**1.2. Viability of Photovoltaics.** Finding a source of energy to meet our 4087 TWh demand for electricity is not difficult. Through a brief derivation, we can calculate the total solar radiation striking the Earth, and show that this power would be enough to meet our demands.[4] To determine the total power of the solar radiation striking the Earth's surface, we can treat the Sun as a blackbody in terms of its emission of electromagnetic radiation. This allows us to apply the Stefan-Boltzmann law with an emissivity $\epsilon = 1$, which states

$$P = A\sigma T^4 \tag{1.1}$$

With $\sigma = (2\pi^5 k^4)/(15c^2h^3) = 5.67 \times 10^{-8}\ \mathrm{W/(m^2K^4)}$. For this, we can calculate the surface area of the sun by approximating it as a sphere, yielding the equation

$$A_s = 4\pi {R_s}^2 \tag{1.2}$$

Using $R_s = 6.963 \times 10^8$ m for the radius of the Sun, and $T_s = 5777$ K for the temperature of the Sun, we can write the power output of the Sun as

$$P_s = A_s\sigma {T_s}^4 \tag{1.3}$$

This energy spreads out uniformly in 3-D space, and thus must be equally distributed along the surface area of a sphere with a radius equal to the distance between the Earth and the Sun. In this way, we can consider the intensity to be written as

$$I = \frac{P_s}{4\pi {R_{se}}^2} \tag{1.4}$$

$$I = \frac{{R_s}^2\sigma {T_s}^4}{{R_{se}}^2} \tag{1.5}$$

Using $R_{se} = 1.496 \times 10^{11}$ m to represent the distance between the Sun and Earth. Plugging in the values for the variables as listed above, we are able to calculate the intensity of the Sun's radiation at Earth:

$$I = \frac{(6.963 \times 10^{8}\ \mathrm{m})^2 \left(\frac{2\pi^5 k^4}{15c^2h^3}\right)(5777\ \mathrm{K})^4}{(1.496 \times 10^{11}\ \mathrm{m})^2} = 1368.11\ \frac{\mathrm{W}}{\mathrm{m}^2} \tag{1.6}$$

Therefore, in each square meter of sunlight intercepted by the Earth, up to 1368.11 W of power is available. This is before considering any interactions with the atmosphere. These interactions vary depending on weather conditions and latitude, either absorbing the radiation or reflecting it back into space, preventing it from reaching Earth's surface. For this brief theoretical analysis, it will become evident that these interactions can effectively be ignored due to the sheer magnitude of power available. To illustrate this, we can arbitrarily reduce the intensity of solar radiation by a generous 60%, to a value of $I_{surf} = 547.24\ \mathrm{W/m^2}$. This value is lower than most estimates for the intensity of the Sun's radiation at Earth's surface.[5] In Chapter 3, for example, we will use the AM 1.5 spectrum, which has a total intensity value of 887.65 $\mathrm{W/m^2}$ at the surface. Taking Earth's radius to be $R_e = 6.371 \times 10^6$ m, the total power available at the Earth's surface is

$$P_e = I_{surf}\pi {R_e}^2 = 6.978 \times 10^{16}\ \mathrm{W} = 69780\ \mathrm{TW} \tag{1.7}$$

This is a truly massive value for power, and is effectively unimaginable. To illustrate this absurdly large value, we can compare it to the power required to meet our entire country's demand for energy, a total of 4087 TWh.

$$t = \frac{4087\ \mathrm{TWh}}{69780\ \mathrm{TW}} = 0.0586\ \mathrm{h} = 210.9\ \mathrm{s} \tag{1.8}$$

Therefore, we would only need to harvest three and a half minutes worth of the energy striking Earth's surface to meet all of our electricity needs for an entire year. Recall that we have already reduced the incoming radiation by 60%, which underestimates the power available at the surface. We will continue underestimating each factor to thoroughly illustrate the viability of solar energy in general. To account for the latitude of the United States, we will use the latitude of Portland, Maine, which reduces the intensity of solar radiation by spreading across a larger area. Using this latitude of 43.6°, our new value for the intensity at the surface is calculated as

$$I_f = I_{surf} \cos 43.6° = 396.30 \ \frac{\mathrm{W}}{\mathrm{m}^2} \tag{1.9}$$

In reality, our generating facilities would be placed in areas with a large amount of solar radiation available, such as in Arizona. Using an approximate conversion efficiency of 15%, and assuming only 5 hours of sunlight is available each day at an intensity of $I_f$, we can now consider the percentage of United States territory that we would need to dedicate to solar panels. At a surface area of 9.834 $\times 10^6$ km$^2$, the percentage of the total area we would need to cover with solar panels would be

$$\frac{A_s}{A_{US}} = \frac{4.087 \times 10^{15} \ \frac{\mathrm{Wh}}{\mathrm{yr}}}{0.15 \ I_f (5 \ \mathrm{h} \times 365 \ \mathrm{d}) 9.834 \times 10^{12} \ \mathrm{m}^2} = 0.0038 = 0.38\% \tag{1.10}$$

Incredibly, we would only need to cover 0.38% of the country's territory with solar panels. Recall that we are using an underestimated value for the intensity of solar radiation at the surface, the average amount of sunlight per day, and the effect of latitude on the generating facility. This would indicate that in reality, we would likely require a much smaller percentage of US territory to be allocated for electricity generation. To demonstrate the feasibility of this task, we have already covered 1.6% of the country's area with paved surfaces, an area the size of the state of Illinois. As will be described in the following sections, real efficiency values are significantly greater, which would also reduce the land use required to meet our country's energy needs entirely with solar power. In this way, it is safe to view solar power as a viable form of producing electricity sustainably for the rest of Earth's existence.

On our planet, around 99% of energy is derived from the sun, with the remainder resulting from the 47 TW of radioactive activity occurring inside the earth[6] and tidal energy. Fossil fuels, which are produced by the fossilized remains of plants and animals that lived millions of years ago, are essentially the Earth's stored solar energy reserves. Plants derive their energy from photosynthesis, and other organisms then consume those plants to fuel their own metabolism. Thus, any energy left over in their remains is simply a stored form of solar energy. Hydropower, which has been used for centuries to do useful work, is driven by evaporation and rain, which the sun is also responsible for. Wind turbines have been used in several applications and are powered by rising warm air, which was heated by the sun. In essence, the human race has been unwittingly relying on solar energy for its entire existence. Now, it is possible for us to bypass the millions of years required to use solar energy as fossil fuel and harvest the energy of the sun directly. The ultimate goal is to economically, sustainably, and efficiently meet the energy needs of the United States through a solar powered generator. In the following analysis in Chapter 5, we will investigate the viability of concentrator photovoltaics to meet those goals.

**1.3. History of Photovoltaics.** Currently, the majority of photovoltaic generating facilities, which convert solar energy directly to electricity, are primarily composed of single-junction cells. Concentrator photovoltaics, on the other hand, are much less widely implemented.

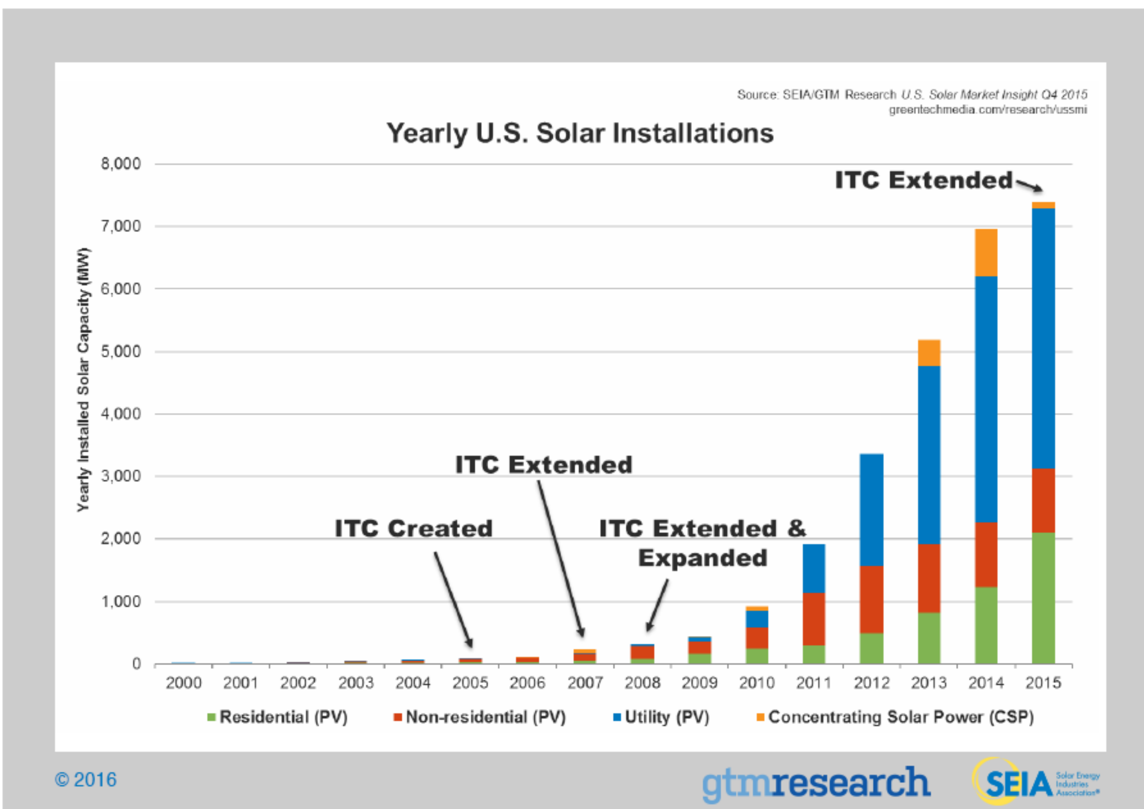


FIG. 1. This figure shows a brief history of solar installations in the United States. Before 2000, solar energy was not a widely implemented source of energy due to low module efficiencies and high installation costs. The “CSP” totals include both CPV and thermal concentrating facilities.

As shown in Fig. 1, it is clear that single junction PV has absolutely dominated the market, in the form of residential, non-residential, and utility applications. There are several factors that have shaped the solar industry in this way. The simplest explanation is the levelized cost of electricity, or LCOE. In single junction photovoltaics, the LCOE is substantially lower, even currently after CPV has seen drastic improvements.[7] Until very recently, the energy portfolio of large utility companies was primarily determined by the companies themselves. This freedom allowed the utilities to choose the cheapest forms of power. Their company values were driven by the consumer's desires, being low costs and reliable energy available every day. In this way, solar was not a good fit.

The market for solar cells is analogous to the causality problem of, "which came first, the chicken or the egg." Because utilities wanted cheap power, fossil fuels and nuclear energy were implemented. This produced a huge demand for these types of power plants. Being in the coal power business involved the development of systems to deliver that electricity as inexpensively as possible. Operations in this category ranged from arranging the logistics to improving the efficiency of the generators. In this way, there was extensive research carried out in developing the best fossil fuel and nuclear generating practices. For example, companies determined that the best way to deliver coal to a power plant was by rail, where the minimal rolling friction provided an incredibly inexpensive means of transportation. Before the year 2000, solar power was expensive and ineffective. However, this didn't necessarily mean that solar was a poor form of electricity generation. These traits defined solar energy because there was essentially zero demand for it on a large scale. Without demand, there was no incentive to make solar power better, because profitability was so distant.

Imagine, for a moment, that solar power and coal had their roles in history swapped. In order for coal to reach the price point it exists at today, a huge investment of capital would be necessary. First, the efficiency of the power plants would need to be optimized to not waste the precious coal, which would need to be expensively trucked in without the existing infrastructure. Then, thousands of miles of railroad tracks would need to be laid, along with the manufacturing of railcars and engines to go with them. This only examines two problems in that scenario. The position that solar power was in twenty years ago is similar. Demand for solar has only recently increased, which has allowed solar to develop to the point where it is beginning to be truly cost-competitive with coal and other methods of electricity generation.

Now, we can extend this same thought experiment to single junction photovoltaics and concentrator photovoltaics. At the time that the demand for solar power was increasing, single junction photovoltaics were naturally the most inexpensive alternative. They were very simple and it was easy to quickly reduce the costs of manufacturing as a result of their simplicity. Concentrator photovoltaics require significantly more time and effort to develop, resulting from complexities in dual axis tracking and the large size of each module. Therefore, it was single junction photovoltaics that were chosen by the free market over concentrator photovoltaics. This is the reason that initially, single junction cells dominated the market, as seen in Fig. 1. It is not until 2010 that CSP becomes visible in Fig. 1, and even then it only occupies a sliver of the total photovoltaic additions to the grid. This is because more financial support has come to solar power, with a total global investment of $80.9 billion.[8] Some of these funds were naturally allocated to investigating alternative methods of collecting solar energy in hopes that their cost would be lowered beyond single junction photovoltaics. Although investments exist to improve CPV, the scars from the past still remain. In this way, the higher historic costs initiated a cycle of low demand and low funding for concentrator photovoltaics.

Another factor that played into the free market's preference for single junction photovoltaic cells is the complexity of concentrator photovoltaics. In order for the concentrating lens to focus light onto the multi-junction cell, the incident solar radiation must be normal to the face of the module. Due to the combination of the Earth's rotation, axial tilt, and elliptical orbit around the Sun, the modules must be mounted on a very sophisticated foundation. They must be able to move in such a way that meets the unique challenge of each day. A system of this nature requires computer software to operate the motors on each module. For this reason, in order to decrease costs, it is desirable to make the modules larger in order to decrease the number of hydraulic motors required to move the large array of lenses and cells. However, as the size of the system increases, its vulnerability to damage in high wind conditions also increases. Therefore, there is a limit on how large the modules can be made before price begins to increase again.

The requirement for normal radiation to be incident on the cell also poses some efficiency losses in the realistic application of the technology. As the sunlight enters the Earth's atmosphere, it is scattered. Our atmosphere primarily scatters light with wavelengths corresponding to blue light, which is why the sky appears blue.

Fig. 2. This is an image of the Universal Module from Arzon Solar LLC in a utility scale generating facility. This is a concentrator photovoltaic system, implementing lenses that focus light onto small multi-junction solar cells. Note the large size and height of each module, as opposed to single junction solar cells that are much smaller. Also, compare the blue sky to Fig. 3.

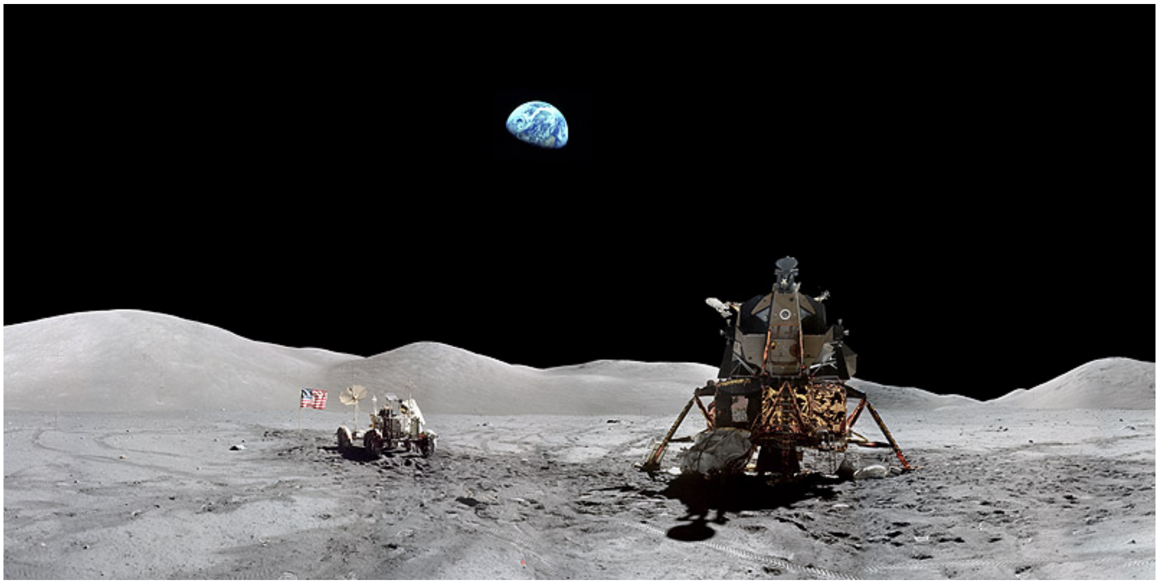

FIG. 3. Earth as viewed from the surface of the Moon. Note that the sky is completely black, besides our beautiful Earth, resulting from the Moon's lack of an atmosphere to scatter light, as exists on Earth. Compare this to the blue sky in Fig. 2.

This scattered light still strikes the Earth's surface, as attested to by the fact that we can see the blue sky. The blue light is interacting with our retina to produce the blue image. Therefore, this scattered light is available for electricity production on the surface. An efficiency calculation would include the energy carried by these photons. In this way, any solar converter that is unable to collect this energy must accept this inefficiency. The wavelength of blue light is approximately 475 nm. However, let us examine green light, which carries less energy than blue light. The wavelength of green light is approximately 510 nm. Since the energy of a photon is defined as $E = (hc)/\lambda$, using $h = 4.135 \times 10^{-15}$ eV $\cdot$ s and $c = 3 \times 10^{8}$ m/s, we can calculate the energy of a green photon, which carries less energy than a blue photon:

$$E = \frac{(4.135 \times 10^{-15}\ \text{eV} \cdot \text{s})(3 \times 10^{8}\ \ \text{m/s})}{(510 \times 10^{-9}\ \text{m})} = 2.43\ \text{eV} \tag{1.11}$$

By proving green light has sufficient energy, we know that blue light will also have sufficient energy. In a silicon solar cell, which is typically used in single junction photovoltaic applications, the band gap energy is 1.1 eV. In order to produce electricity, the energy of the incident photon must carry energy equal to or greater than the band gap energy. As discussed, the light scattered by the atmosphere reaches the surface and is available for electricity generation. In a single junction photovoltaic generator without a concentrating lens, the angle of incidence of a photon has no effect on the production of hole-electron pairs, and thus no effect on the electricity output. However, in concentrator photovoltaics, there is an irreparable problem with collecting the scattered light. The system relies on a lens to concentrate the incoming solar radiation, and in order for light to strike the solar cell behind the lens, it must be near normal to the plane of the face of the module. It does not need to be exactly normal, as discussed in section 3.2. If you consider the thought experiment presented in section 3.2, unless the cell is under full concentration, some of the sky would be “visible” from the perspective of the cell, and thus would be striking the cell to produce power. However, much of the scattered light does not strike the cell. This is one limiting factor on the efficiency which is not typically accounted for, and that cannot be optimized. This provides a fundamental advantage in the application of single junction photovoltaics, which will tend to perform better than their concentrator photovoltaic counterparts as a result of this scattering phenomenon.

In addition to this loss of efficiency, concentrator photovoltaics also must endure several other factors that decrease electricity output. There are optical losses associated with the reflection of light off of the lens. This will be examined in further detail in Chapter 4. Also, depending on the design of the system and the materials used, concentrating solar energy up to 1000x unsurprisingly increases cell temperatures. As will be discussed in Chapter 2, an increase in temperature results in a decrease in the maximum theoretical efficiency of a solar converter. As a result, there is a decrease in the maximum theoretical efficiency of a p-n junction solar cell as well.

Concentrator photovoltaics were not popular as a result of these various factors, which reduced their attractiveness to utilities. However, as illustrated in Fig. 1, they are beginning to experience some market penetration. Not only has their efficiency improved drastically, but their LCOE has also decreased to between $0.08 per kWh and $0.15 per kWh[7]. At this point, they are beginning to be competitive economically, especially after incentives. This is a recent development in concentrator photovoltaics. As a result, very little analysis of their applications exist when compared to single junction photovoltaics and other forms of electricity generation. The primary purpose of this paper is to rigorously analyze the viability of concentrator photovoltaics in the United States, and assess its ability to assist in the conversion of our energy systems to sustainability.

**1.4. Methodology.** In order to evaluate the viability of concentrator photovoltaics, we must compare various figures to an established baseline.

Naturally, this baseline will be defined as single junction photovoltaics. In this way, we can easily examine the functionality of the two generating systems and determine the scenarios in which one outperforms the other. At this baseline, a range of measurements can be made. In this analysis, the following components will be compared:

- Maximum theoretical efficiencies, as a way to judge the potential for efficiency improvements
- Efficiency on a land use basis (useful in considering large scale applications)
    - Land use/environmental impact
- Levelized Cost Of Electricity (LCOE) – the overall cost at which electricity is produced

It is difficult to define weights for the importance of each of these categories. Rather, they tend to work together, and are overall measured by the LCOE. However, it is instructive to examine each component, providing insight regarding methods of lowering the LCOE. Especially in the case of cell efficiency, a general upper limit that a given factor can be optimized can be established. Therefore, we will examine each element of the photovoltaic systems and consider the results together to determine the level of viability concentrator photovoltaics have for utility scale electricity production.

# CHAPTER 2

## Detailed Balance Limit Efficiency for Single p-n Junctions

A widely cited source on the subject of theoretical maximum efficiencies is the detailed balance limit for p-n junction solar cells described by William Shockley and Hans Queisser. This derivation examines the most fundamental properties of a single junction photovoltaic cell, developing a maximum conversion rate from electromagnetic radiation from the sun to a potential difference and thus electrical energy from the solar cell. It consciously neglects many factors, such as optical losses or changes in the solar spectrum, in order to determine the limit imposed by the p-n junction itself. Although the value for efficiency reached in the paper is in fact unattainable experimentally due to realistic limiting factors, it serves as a glass ceiling past which no single junction solar cell could ever reach.

**2.1. Significance.** The significance of the detailed balance limit is based in its insight in estimating the potential for improvement in solar cells by comparing the result to efficiencies that are currently attainable experimentally. The authors illustrate the demand for such a limit in the field of photovoltaics using an example of the maximum thermodynamic efficiency of a solar converter on Earth. Using the Carnot Cycle, the theoretical maximum efficiency at which any system can convert thermal energy into work is given by

$$\eta = \frac{T_H - T_C}{T_H} \tag{2.1}$$

For a solar converter on Earth's surface, we can take $T_C = 300$ K and $T_H = 5777$ K, yielding the efficiency

$$\eta = \frac{5777\text{ K} - 300\text{ K}}{5777\text{ K}} = 0.948 = 94.8\% \tag{2.2}$$

This implies that a solar converter can harvest nearly all of the Sun's energy. With single junction cells hovering between 15% to 20% efficient, it would seem that p-n junctions are vastly underperforming their potential, and have a massive potential for improvement. Without the detailed balance limit, researchers would be searching for massive improvements, possibly casting aside great discoveries that would improve efficiency by a few percent. However, with the detailed balance limit efficiency of 30% for silicon cells, it becomes clear that an improvement of a few percentage points would indeed be a very valuable discovery in the field of photovoltaics.

**2.2 Ultimate Efficiency.** In the detailed balance limit, the efficiency is restricted by a few factors. The ultimate efficiency, $u(x_g)$, is determined by the mechanism that a p-n junction relies on to produce electricity. In a p-n junction, there is a band gap energy of $E_g$, which varies by the material used to create the solar cell. If the energy of an incoming photon, $E = h\nu$, equals or surpasses the energy of the band gap, the photon will be absorbed, and will create a hole-electron pair across the band gap. Note, however, that this potential difference is $V_g = E_g/q_e$, not $V = E/q_e$. In the case that $E > E_g$, the hole-electron pair created by the incident photon will carry less energy than the photon itself carried. This property of p-n junctions introduces two ways in which energy incident on the solar cell is lost in the conversion process. Initially, any photon with an energy less than the band gap, or $E < E_g$, will not produce a hole-electron pair at all. This means that the energy carried in frequencies of light $\nu < \nu_g$ is inaccessible to a single junction solar cell. After a photon has met the criteria of having frequency $\nu \geq \nu_g$, there are further losses to be accounted for. Since the incident photon had energy $E \geq E_{g,}$ but only produced a potential difference of $V_g$, each photon results in an additional loss of energy $E - E_g$. To illustrate these two effects mathematically, we must begin by filtering out all photons with a frequency $\nu < \nu_g$. By integrating Plank's Law across all frequencies emitted by the Sun, we can write the total power output of the Sun as

$$P_s = \frac{2\pi h}{c^2} \int_0^\infty \frac{\nu^3}{e^{(h\nu/kT_s)} - 1} \mathrm{d}\nu \tag{2.3}$$

After dividing this equation by the energy of each photon $h\nu$, we can limit the lower bound to $\nu_g$ to write an equation for the number of photons with frequency $\nu \geq \nu_g$

$$Q_s = \frac{2\pi}{c^2} \int_{\nu_g}^\infty \frac{\nu^2}{e^{(h\nu/kT_s)} - 1} \mathrm{d}\nu \tag{2.4}$$

Using $x = h\nu/kT_s$, $x_g = h\nu_g/kT_s$ and $\mathrm{d}x = (h\,\mathrm{d}\nu)/kT_s$, we can simplify the integral to write the final equation for the number of usable photons as

$$Q_s = \left(\frac{2\pi}{c^2}\right)\left(\frac{kT_s}{h}\right)^3 \int_{x_g}^\infty \frac{x^2}{e^x - 1} \mathrm{d}x \tag{2.5}$$

This equation involves a Riemann zeta function, and cannot be written in a functional form. However, the integral can be evaluated numerically for a given lower bound $x_g$. Because of this element involved in the ultimate efficiency, when writing the general detailed balance limit efficiency of $\eta(x_g, x_c, t_s, f)$, we will see $u(x_g)$ appear explicitly in the equation for $\eta$.

After determining the number of photons that will produce hole-electron pairs, we can proceed to the next limiting factor, which is that each photon can only yield the band gap energy. Simply multiplying the number of photons by the band gap energy yields the total energy that can be converted to electricity, ignoring all other factors. Our equation for the power output of the cell is

$$P_{out} = Q_s E_g = h\nu_g Q_s \tag{2.6}$$

Therefore, we can derive the ultimate efficiency for a single p-n junction solar cell to be

$$u(x_g) = \frac{P_{out}}{P_s} = \frac{h\nu_g Q_s}{P_s} \tag{2.7}$$

We can obtain a cleaner expression for $P_s$ by again using $x = h\nu/kT_s$ , producing

$$P_s = \left(\frac{2\pi(kT_s)^4}{h^3c^2}\right)\int_0^\infty \frac{x^3}{e^x - 1}\mathrm{d}x \tag{2.8}$$

$$P_s = \left(\frac{2\pi(kT_s)^4}{h^3c^2}\right)\left(\frac{\pi^4}{15}\right) \tag{2.9}$$

$$P_s = \frac{2\pi^5(kT_s)^4}{h^3c^2} \tag{2.10}$$

Using our simplified equation for the magnitude of power striking the cell, we can finish the equation for the ultimate efficiency as

$$u(x_g) = \frac{15x_g}{\pi^4}\int_{x_g}^{\infty}\frac{x^2}{e^x - 1}\mathrm{d}x \tag{2.11}$$

Although this is not the complete detailed balance limit, we can still interpret its value as the limit imposed by the hole-electron pair production mechanism in a semiconductor. For instance, let's consider the case of silicon, which has a band gap energy of 1.1 eV. This corresponds to a band gap frequency $\nu_g = 2.66 \times 10^{14}$ Hz which gives us $x_g = 2.2097$. Plugging this into the ultimate efficiency, by evaluating the Riemann zeta function we find that $u(x_g) = 0.4386 = 43.86\%$.

However, as will be proven in the Chapter 3 derivations for ultimate efficiency, this value is potentially inaccurate. This inaccuracy is a result of the differing spectrums used to calculate the ultimate efficiency. The shape of the radiation's distribution plays a critical role in determining the ultimate efficiency. For example, if the distribution consisted of a spike at the band gap wavelength, the ultimate efficiency would be 100% (Note that this does not imply a solar cell could exceed the Carnot Cycle limit, because this is only the ultimate efficiency). All photons would produce hole-electron pairs, and none of their energy would be wasted. In Chapter 3, the AM 1.5 spectrum is used, which is a more accurate depiction of the solar radiation available at the surface. As a result of using this spectrum, instead of using a blackbody spectrum with the temperature of the sun, the ultimate efficiency is found to be 50.42% for silicon p-n junctions. In this derivation, this will be the first term in the detailed balance limit. As a result of using this term, the detailed balance limit efficiency will be increased by around 4.5%.

**2.3. Current-Voltage Relationship.** The next limit is imposed by the conservation of charge. This requires that hole-electron pairs be removed at the same rate as they are produced. Without this limit, it would be possible for the p-n junction to “run out” of hole-electron pairs. At this point, the solar cell would cease to function. The central motivation for this constraint, however, is the fact that charge cannot be created or destroyed. It is not possible for the solar cell to be a net source of electrons, nor is it possible for the cell to remove electrons from the system.

There are five processes by which hole electron pairs are created and eliminated. We have already discussed how hole-electron pairs are produced by incoming solar radiation. Therefore, the number of hole-electron pairs produced will involve $Q_s$. In addition, we mentioned that the hole-electron pairs produce a current across a voltage of $V_g$, which eliminates hole electron pairs at a rate $I/q_e$. To introduce the other three mechanisms by which hole-electron pairs are created and destroyed, some background on recombination is required.

There are three types of recombination that must be considered. In radiative recombination, a photon is emitted as a hole-electron pair is eliminated. Shockley-Read-Hall generation and recombination occurs when an electron in the semiconductor is trapped in an irregularity in the material, and in a second step is released and recombines with the hole it created. Finally, there is Auger recombination, in which the energy stored in the hole-electron pair is converted to thermal energy in the solar cell by transferring its energy to a third carrier in the material. With these five mechanisms, we can write an expression for the rate of creation and elimination of hole-electron pairs. As stated earlier, we know they must collectively equal 0. That equation is

$$F_s - F_c(V) + R(0) - R(V) - \frac{I}{q_e} = 0 \tag{2.12}$$

It is clear that $F_s$, which represents the rate at which hole-electron pairs are produced by the Sun, must involve $Q_s$, the number of photons above the bang gap energy that strike a given area. Therefore, we must multiply $Q_s$ by the area of the cell. Also, we must consider that the incoming solar radiation originates from a small area in space. This geometrical factor can be written as $f_\omega = (R_e/R_{se})/2$. Together, the expression for the rate at which hole-electron pairs are produced by the incident solar radiation is

$$F_s = f_\omega A_c Q_s \tag{2.13}$$

Let us also define the blackbody radiation originating from the cell as $F_{c0} = 2A_cQ_c$. With this new term, we can rewrite the current-voltage relationship as

$$F_s - F_{c0} + \left(F_{c0} - F_c(V) + R(0) - R(V)\right) - \frac{I}{q_e} = 0 \tag{2.14}$$

Next, we can define the factor $f_c$ as the fraction of recombination that is radiative:

$$f_c = \frac{F_{c0} - F_c(V)}{\left(F_{c0} - F_c(V) + R(0) - R(V)\right)} \tag{2.15}$$

It will be more useful to rewrite Eq. (2.15) in terms of the denominator, in order to more easily manipulate the current-voltage relationship:

$$\frac{F_{c0} - F_c(V)}{f_c} = \left(F_{c0} - F_c(V) + R(0) - R(V)\right) \tag{2.16}$$

Now, solving Eq. (2.14) for the current $I$ yields

$$I = q_e[(F_s - F_{c0}) + F_{c0} - F_c(V) + R(0) - R(V)] \tag{2.17}$$

Also, we need to describe $F_c(V)$, the radiative recombination in the cell. At thermal equilibrium, if we define $V_c = kT_c/q_e$, then $V = V_c$, and we have $F_c(V) = F_{c0}$. In addition, the rate of radiative recombination increases exponentially with voltage.[9] Therefore, the equation for the radiative recombination is

$$F_c(V) = F_{c0}e^{V/V_c} \tag{2.18}$$

Applying our result from Eq. (2.16) after distributing $q_e$ yields

$$I = q_e(F_s - F_{c0}) + q_e\left(\frac{F_{c0} - F_c(V)}{f_c}\right) \tag{2.19}$$

Using Eq. (2.18) for the radiative recombination, we can rewrite the final current-voltage relationship as

$$I = q_e(F_s - F_{c0}) + \frac{q_e F_{c0}}{f_c}\left(1 - e^{V/V_c}\right) \tag{2.20}$$

At this point, we are able to define the short circuit current and the open circuit voltage. First, we can find the short circuit current by setting $V = 0$. This only changes the second term, which goes to zero as a result of the exponential. Therefore, the short circuit current is

$$I_{SC} = q_e(F_s - F_{c0}) \tag{2.21}$$

However, $F_{c0}$ is very small compared to $F_s$, as a result of the very minimal blackbody radiation emitted by the cell itself. This allows the following approximation:

$$I_{SC} = q_e(F_s - F_{c0}) \cong q_e F_s \tag{2.22}$$

Now, we can solve for the open circuit voltage by setting the current to be zero. This is

$$0 = q_e(F_s - F_{c0}) + \frac{q_e F_{c0}}{f_c}\left(1 - e^{V_{OC}/V_c}\right) \tag{2.23}$$

This equation can be further simplified by letting $I_0 = q_e F_{c0}/f_c$. After rearranging and applying our result from Eq. (2.22), we can reach

$$e^{\frac{V_{OC}}{V_c}} = \frac{I_{SC}}{I_0} + 1 \tag{2.24}$$

$$V_{OC} = V_c \ln\left(\frac{I_{SC}}{I_0} + 1\right) \tag{2.25}$$

Using our previous definitions of $I_{SC}$ in Eq. (2.22) and $I_0$, we can further simplify the expression for the open circuit voltage to

$$V_{OC} = V_c \ln\left(\frac{F_s\, f_c}{F_{c0}} - f_c + 1\right) \tag{2.26}$$

Plugging in our definitions for $F_s$ and $F_{c0}$ gives us

$$V_{OC} = V_c \ln\left(\frac{A_c f_\omega Q_s f_c}{2 A_c Q_c} - f_c + 1\right) \tag{2.27}$$

Again, we can make another approximation because the first term is much larger than the second and third terms, since $Q_s \gg Q_c$. Applying this approximation and simplifying the fraction yields

$$V_{OC} \cong V_c \ln\left(\frac{fQ_s}{Q_c}\right) \tag{2.28}$$

Where $f = f_c f_\omega t_s/2t_c$. In theory, the maximum voltage of the cell cannot exceed $V_g$. This is significant because the power delivered by the cell is a function of the voltage, in the form $P = IV$. Therefore, we can build a second limiting efficiency. This will not exceed 100%, because it will always be true that $V_{OC} \leq V_g$

$$v(x_g, x_c, f) = \frac{V_{OC}}{V_g} \tag{2.29}$$

**2.4. Impedance Matching Factor.** Now, the last component of the efficiency is given by the impedance matching factor *m*, which is the ratio between the maximum power and the nominal power. The expression for *m* is

$$m = \frac{P_{max}}{I_{SC}V_{OC}} \tag{2.30}$$

As mentioned above, the equation for power is $P = IV$. Therefore, the following equation can be solved to determine the maximum power:

$$\frac{\mathrm{d}(IV)}{\mathrm{d}V} = 0 \tag{2.31}$$

To obtain a usable expression for *I*, we must manipulate Eq. (2.25) to suit our needs:

$$I_0 e^{\frac{V_{OC}}{V_c}} = I_{SC} + I_0 \tag{2.32}$$

Next, after distributing $I_0$ in Eq. (2.20), we can use Eq. (2.32) with Eq. (2.21) to write

$$I = I_0\left(e^{\frac{V_{OC}}{V_c}} - e^{\frac{V}{V_c}}\right) \tag{2.33}$$

Now, we can plug Eq. (2.33) into Eq. (2.31):

$$\frac{\mathrm{d}}{\mathrm{d}V}\left[I_0 V\left(e^{\frac{V_{OC}}{V_c}} - e^{\frac{V}{V_c}}\right)\right] = 0 \tag{2.34}$$

After taking this derivative, we see that $I_0$ drops out of the equation because it is constant with respect to $V$, leaving us with

$$e^{\frac{V_{OC}}{V_c}} - \left(\frac{V}{V_c} - 1\right)e^{\frac{V}{V_c}} = 0 \tag{2.35}$$

By letting $z_m = V_{max}/V_c$ and $z_{OC} = V_{OC}/V_c$, we can more cleanly write Eq. (2.35) and solve for $z_{OC}$ by taking the natural logarithm of both sides of the equation, yielding

$$z_{OC} = z_m + \ln(z_m + 1) \tag{2.36}$$

Now, we can return to Eq. (2.30) in order to finalize our expression for the impedance factor

$$m\left(\frac{v x_g}{x_c}\right) = \frac{{z_m}^2(z_m + \ln(1 + z_m))}{(1 + z_m - e^{-z_m})} \tag{2.37}$$

**2.5. Maximum Theoretical Efficiency for p-n Junction.** Finally, we are ready to write the detailed balance limit efficiency, which is defined as the impedance matching factor divided by the incoming power.

$$\eta(x_g, x_c, f) = \frac{m(v x_g/x_c)}{P_{inc}} \tag{2.38}$$

The incoming power available to the cell is

$$P_{inc} = f_\omega A_c P_s \tag{2.39}$$

However, as defined in Eq. (2.7), the incoming power can be written to involve the ultimate efficiency as

$$P_{inc} = \frac{f_\omega A_c h\nu_g Q_s}{u(x_g)} \quad (2.40)$$

Therefore, the impedance matching factor as defined in Eq. (2.38) divided by the incoming power defined in Eq. (2.40) yields the detailed balance limit of

$$\eta(x_g, x_c, f) = u(x_g)v(x_g, x_c, f)m(vx_g/x_c) \quad (2.41)$$

Using a blackbody spectrum to represent the Sun's radiation, and a band gap energy of 1.1 eV for silicon p-n junctions, the detailed balance limit comes out to 29.27%. Shockley and Queisser cite a value of 30%, and the discrepancy here comes from the differences in values chosen for the temperature of the Sun. For my derivations, I have used 5777 K, and in their paper they used 6000 K. This substantial difference slightly increases the efficiency. However, when bypassing this by using the AM 1.5 spectrum for the ultimate efficiency, which is a more accurate representation of the Sun's radiation, the detailed balance limit increases to 33.65%. This effect of increased theoretical efficiency as a result of using the AM 1.5 spectrum was recently documented for many band gap energies[10] due to the efficiency's unique dependence on the band gap energy. For a band gap of 1.1 eV, the efficiency found in that paper was 32.2%.

However, this is not to say that the maximum efficiency of solar energy to electricity is necessarily capped at the detailed balance limit. As we will see with multi-junction photovoltaic cells, this efficiency can be surpassed. This limit only applies to a solar generator that converts solar energy directly to electricity through a p-n junction. It does not constrain the harvesting of solar energy in general.

The easiest way to visualize this is by examining the ultimate efficiency. As discussed in the derivation, this part of the detailed balance limit results from a p-n junction's inability to convert lower energy photons into electricity. Yet on our planet, there exist several materials that are very "black," which is to say that they strongly absorb all visible wavelengths of light. The problem is that this energy is converted into heat, not electricity. Demand for solar electricity has only recently increased, and as a result the technologies available are limited. It is very possible that a more efficient mechanism exists to convert solar radiation to electricity, and that humans have not discovered it. As an example, imagine deploying a p-n junction with a variable band gap, with a sensor that could communicate the energy density of the incoming photons. It may be possible for the band gap to be adjusted according to which photons happen to be striking the surface in a given instant, in order to maximize the efficiency for specific photons that carry the most potential electricity. This would most likely violate Einstein's theory of Special Relativity, as a result of information being communicated ahead of the light itself so that the band gap could be adjusted. However, this thought experiment still serves the purpose of imagining what new technologies may be developed in the future, and how they could manage to exceed the detailed balance efficiency. It is also instructive to examine the efficiency of multi-junction photovoltaic cells.

# CHAPTER 3

## Limiting Efficiency in Multi-Junction Solar Cells

In a multi-junction cell, there are layers of p-n junctions in descending order of band gap energies. In this derivation, it is assumed that the p-n junction is transparent for photons that carry less than the band gap energy. This allows photons that normally cannot be used by the first p-n junction to possibly produce a voltage in the next two layers of p-n junctions. In order to extend the detailed balance limit to multi-junction cells, it is easier to develop the energy lost on a per photon basis. In this way, the intrinsic losses encountered along the way can be effectively visualized and easily calculated.

**3.1. Air Mass 1.5 Spectrum (AM 1.5).** For this calculation, it is more accurate to use the air mass 1.5 solar spectrum. In the detailed balance limit for single junction cells, we approximated the Sun as a blackbody with a temperature of $T_s = 5777$ K. This approximation was relevant, because any absorption occurring for frequencies lower than the band gap energy, or wavelengths greater than the band gap wavelength, would have no effect on the efficiency. For example, in silicon solar cells the band gap energy is 1.1 eV. Since $\lambda = hc/E$, we can calculate the exact band gap wavelength:

$$\lambda_g = \frac{hc}{E_g} = \frac{(4.135 \times 10^{-15}\ \text{eV} \cdot \text{s})\left(3 \times 10^8\ \frac{\text{m}}{\text{s}}\right)}{(1.1\ \text{eV})} = 1127.7\ \text{nm} \qquad (3.1)$$

As shown in Fig. 4, the majority of the absorption in the AM 1.5 spectrum occurs at wavelengths greater than the band gap wavelength. This is represented by the wide valleys, which deviate from the trend of a blackbody distribution for a radiating body at the Sun's temperature. Therefore, our approximation of the Sun as a blackbody at $T_s = 5777$ K in the detailed balance limit for single p-n junctions was valid. However, as shown by Eq. (3.6), it did have a noticeable margin of error associated with it.

Now, on the other hand, using the AM 1.5 spectrum is necessary in order to make an accurate calculation of efficiency. Using the blackbody spectrum would cause an overestimation of both power available and delivered by the cell. However, they may not balance out. Although this is only a theoretical calculation, it is still important to be as accurate as possible. Underestimating the efficiency would cause concentrator photovoltaics to appear less appealing than they would be in their application, because they would possess a greater potential to improve than predicted. Overestimating the efficiency would also be counterproductive, for the same reason discussed in Chapter 1. An overestimated efficiency could cause researchers to pass up or ignore an improvement of a few tenths of a percentage point if they believed the ceiling was much farther than that. However, in reality this seemingly small improvement would actually be a very useful discovery. In light of the consequences of inaccurately calculating the theoretical maximum efficiency, it is best to use the AM 1.5 spectrum in this calculation.

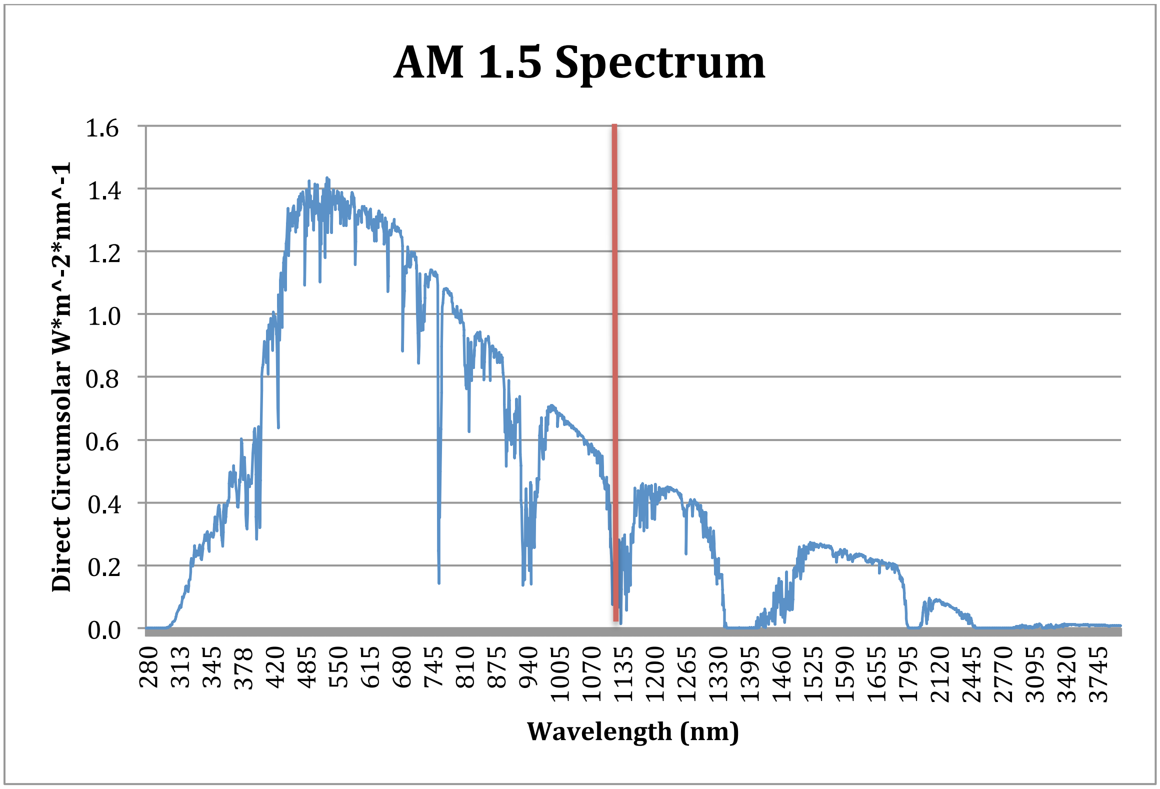


FIG. 4. The air mass 1.5 spectrum accounts for the effect of Earth's atmosphere on the incoming solar radiation. The valleys correspond to the absorption bands of specific molecules in our atmosphere, such as water vapor. The orange line represents the band-gap wavelength of silicon. Photons that lie on the right side of this line are inaccessible to a silicon p-n junction, while the photons on the left will produce a hole-electron pair.

**3.2. Detailed Balance Limit.** The following derivation uses methods from C. H. Henry's paper on terrestrial solar cells.[9] Recall that a multi-junction cell is essentially two or more single-junctions laid on top of one another. Therefore, a similar ultimate efficiency as defined in Eq. (2.7) must be accounted for. In effect, this cuts off a region of the AM spectrum, as shown in Fig. 4. As a result of the band gap energy, the lower energy photons are unavailable for hole-electron pair production. However, as discussed in Chapter 2, the photons exceeding the band gap energy are unable to convert their excess energy into output power. Therefore, this is another source of loss, and decreases the maximum possible theoretical efficiency. This ultimate efficiency is relatively easy to calculate using Excel. The total incoming power is calculated simply by adding all the power values across all wavelengths, presenting a value of

$$P_{AM1.5} = 887.65 \ \frac{\mathrm{W}}{\mathrm{m}^2} \tag{3.2}$$

Next, we need to find the number of photons striking the cell with $E \geq E_g$. To find this value in Excel, we first need to develop an expression for the number of photons at a given wavelength. The energy of a photon is $E = hc/\lambda$. Since the AM 1.5 spectrum provides us with the total energy associated with a given wavelength, the number of photons with energy $E$ incident on the cell is

$$Q_{AM1.5} = \frac{P}{E} \tag{3.3}$$

Where $P$ is the power at a given wavelength. Since both the power and energy are functions of $\lambda$, we need to numerically integrate this equation for $\lambda < \lambda_g$. This yields

$$Q_{AM1.5} = 2.54 \times 10^{21} \ \frac{\mathrm{photons}}{\mathrm{s} \cdot \mathrm{m}^2} \tag{3.4}$$

Since each of these photons produces a hole-electron pair, the ultimate max power output of a silicon cell is

$$P_{max} = Q_{AM1.5} E_g = 447.55 \ \frac{\mathrm{W}}{\mathrm{m}^2} \tag{3.5}$$

Therefore, the ultimate efficiency in a silicon cell is

$$u_{AM1.5} = \frac{P_{max}}{P_{AM1.5}} = \frac{447.55\,\frac{\mathrm{W}}{\mathrm{m}^2}}{887.65\,\frac{\mathrm{W}}{\mathrm{m}^2}} = .5042 = 50.42\% \tag{3.6}$$

Interestingly, this differs substantially from our value obtained in Chapter 2, which was 43.86%. This results from using the AM 1.5 spectrum instead of relying on a blackbody distribution from a 5777 K object to replicate the spectrum produced by the sun.

However, as demonstrated in the detailed balance limit for single p-n junctions in Chapter 2, recombination must be taken into account in order to find the correct theoretical maximum efficiency. For the ultimate efficiency, we assumed that a photon with $E \geq E_g$ produced an amount $E_g$ of energy in Eq. (3.5). However, recombination reduces this energy produced. Let us call the actual work done per photon $W$. To define $W$, it is useful to use charge densities to describe the work done. Evidently, the work done per photon would be the total work done divided by the number of photons, so we have

$$W = \frac{J_{max} V_{max}}{Q_{AM1.5}} \tag{3.7}$$

Next, we must account for the effect of recombination on the cell's efficiency. Recombination reduces the maximum voltage and current, and thus the power output of the cell. Therefore, we can simply include recombination as the sum of charge densities. Defining $J_{ph} = q_e Q_{AM1.5}$, the net charge density incident on the solar cell is

$$J = J_{ph} - J_{rec} \tag{3.8}$$

Following a procedure similar to Chapter 2, we can determine the open-circuit voltage, which corresponds to setting $J = 0$. Taking the recombination current to be $J_{rec} = Ae^{(q_e V - E_g)/kT}$ and $J_{ph}$ to be Eq. (3.4) times the charge of an electron $q_e$, Eq. (3.8) becomes

$$J = q_e Q_{AM1.5} - Ae^{\frac{(q_e V - E_g)}{kT}} \tag{3.9}$$

As before, we can calculate the open circuit voltage by setting the current equal to zero, which corresponds to $J = 0$. Solving this for the open circuit voltage yields

$$V_{OC} = \frac{1}{q_e}\left(E_g - kT\ln\left(\frac{A}{q_e Q_{AM1.5}}\right)\right) \tag{3.10}$$

In order to find the maximum power delivered by the cell, we must find the voltage, which must be less than the open circuit voltage, maximizing power. This is found by solving

$$\frac{\mathrm{d}}{\mathrm{d}V}(JV) = 0 \tag{3.11}$$

Solving this equation for the voltage, which is the voltage at which the cell has maximum power, produces

$$V_m = \frac{1}{q_e}\left(E_g - kT\ln\left(\frac{A}{q_e Q_{AM1.5}}\right)\right) - \frac{kT}{q_e}\ln\left(1 + \frac{q_e V_m}{kT}\right) \tag{3.12}$$

Using this maximum voltage in Eq. (3.9) yields

$$J_m = \frac{q_e Q_{AM1.5}}{1 + kT/q_e V_m} \tag{3.13}$$

Finally, taking these results back to the starting point for *W,* Eq. (3.7), we have

$$W \cong E_g - \mathrm{kT}\left[\ln\left(\frac{A}{q_e Q_{AM1.5}}\right) + \ln\left(1 + \frac{q_e V_m}{kT}\right) + 1\right] \tag{3.14}$$

It is at this point that we are able to understand the effect of concentrating sunlight onto the solar cell. This equation for $W$ assumes no concentration of sunlight. Effectively, placing a lens in front of the cell would increase the flux of photons on the cell, and would thus increase $Q_{AM1.5}$ by some concentrating factor $C$. After moving the $C$ term out of the logarithm, it is clear that the concentration would increase the work done per photon by $kT\ln(C)$. This becomes important in Chapter 4, in which the limit on the concentrating factor is discussed.

We can calculate the overall efficiency as

$$\eta = \frac{W}{\langle h\nu_g \rangle} \tag{3.15}$$

As written, evaluating Eq. (3.15) would present the efficiency limit for a silicon solar cell, as a result of using $Q_{AM1.5}$. However, the goal is to determine the efficiency limit for a multi-junction cell, which has multiple band gaps. As explored in Chapter 5, the maximum theoretical efficiency that a multi-junction cell has obtained has a semiconductor structure of GaInP/GaAs/GaInAsP/GaInAs[11]. Their respective band gap energies are 1.87 eV[12], 1.42 eV[12], 1.14 eV[13], and 1 eV[12]. Using Fig. 4, we can see that at 1000 Suns of concentration, this corresponds to respective values of $W$ of 1.56 eV, 1.16 eV, 0.89 eV, and 0.76 eV, producing a maximum efficiency of 56.3%.

**3.3. Maximum Concentration.** In the previous derivation, we examined efficiencies for unaltered sunlight and concentrated sunlight. In the equation for $W$, it was shown that a greater photon flux incident on the cell caused $W$ to increase, which in turn increased the overall efficiency. One mechanism that produces a greater photon flux is a concentrating lens. Because increasing the photon flux corresponds to $kT\ln(C)$, it would seem that the maximum efficiency would be obtained by infinitely concentrating the incoming sunlight. However, a limit on concentrating sunlight exists based on the geometry of the system. The best way to visualize this is a thought experiment. Imagine you are the solar cell, and you are looking into the sky. Without any concentrating lens, the sun would appear to occupy some small percentage of the sky. If you held your thumb up at the length of your arm, it would be able to eclipse the sun. Now, imagine placing a concentrating lens in front of your vision. This is essentially a magnifying glass, and thus the image detected by your eyes would be larger than before. Perhaps at this point you would need your entire hand to block out the Sun. If we continue increasing the power of the lens, the image of the Sun would continue getting larger. At some point, however, the image of the Sun would occupy your entire field of vision. Any further concentration would result in "zooming in" on the image you currently see, and the outer rim of the Sun would escape your field of vision. Human vision spans about 160 degrees, and thus is almost exactly analogous to the photons striking a solar cell, which spans 180 degrees. The "zooming in" that we can picture occurring past maximum concentration corresponds to sunlight that is so concentrated that it crosses from one side of our field of vision to the other, never striking our retina and instead propagating past our ear. Because that sunlight carries energy, any further concentration beyond the point where the Sun begins to occupy our entire field of vision would cause losses in efficiency. Therefore, we must mathematically determine what the optimum concentration factor is in order to maximize the C-factor in the equation for *W,* and thus maximize efficiency.

A simple method of calculating this maximum concentration factor is utilizing conservation of energy. Assuming no optical losses,

$$P_{in} = P_{out} \tag{3.16}$$

From Eq. (1.5), we have

$$P_{in} = \frac{A_{lens}{R_s}^2\sigma{T_s}^4}{{R_{se}}^2} \tag{3.17}$$

In the same way, we can apply Stefan-Boltzmann Law to the absorbing area, yielding

$$P_{out} = A_{abs}\sigma{T_{abs}}^4 \tag{3.18}$$

Next, we can define the concentrating factor as the ratio of the area of the lens to the area of the absorbing area. This is

$$C = \frac{A_{lens}}{A_{abs}} \tag{3.19}$$

By dividing by $P_{out}$ in Eq. (3.16) and plugging in our expressions for power, we can write

$$1 = \frac{A_{lens}{R_s}^2\sigma{T_s}^4}{\left(A_{abs}\sigma{T_{abs}}^4\right){R_{se}}^2} \tag{3.20}$$

In the case that we have maximum concentration, $T_s = T_{abs}$. After simplifying, it is clear that the concentrating factor appears in this equation:

$$1 = \frac{A_{lens}{R_s}^2}{A_{abs}{R_{se}}^2} \tag{3.21}$$

Therefore, we can rewrite this equation to produce a value for the maximum concentrating factor.

$$C_{max} = \left(\frac{R_{se}}{R_s}\right)^2 = \left(\frac{1.496 \times 10^{11}\ \mathrm{m}}{6.963 \times 10^{8}\ \mathrm{m}}\right)^2 = 46160.5 \tag{3.22}$$

Typically, this value is rounded to 46000.[14] Briefly, we can take this value and examine in what way it can increase $W$. Because of the nature of the variability of the efficiency's dependence on $W$, it is difficult to extract the exact possible efficiency increase as a result of using maximum concentration. Referring to Eq. (3.14), we can see $C_{max}$ increases $W$ by a factor of $kT\ \ln(C_{max}) = 0.28$ eV. For a concentration of $1000 = C$, which is more realistic, we see that $W$ is increased by 0.19 eV. Both of these results are substantial, given that the difference between $W$ and the band gap energy typically ranges between 0.4 and 0.5.[9]

# CHAPTER 4

## Optics Applied to Concentrator Photovoltaics

In concentrator photovoltaics, a lens is used to focus light on a multi-junction solar cell, which converts sunlight to electricity more effectively than traditional single junction solar cells. Up until this point, we have ignored the effect of optical losses associated with the lens. As an example of this, in section 3.2 we assumed the power incident on the lens was equal to the power transmitted by the lens. As will be shown in section 4.3, this is not the case. The focus of this chapter will be examining the impact of optics on the maximum theoretical efficiency of concentrator photovoltaics.

**4.1. Index of Refraction.** In order to understand how the lens will interact with the incident electromagnetic waves, a definition of the index of refraction is necessary. From Maxwell's Equations, the speed of light through a medium is

$$v = \frac{1}{\sqrt{\epsilon\mu}} \tag{4.1}$$

The index of refraction is defined as the ratio of the speed of light in a vacuum to its speed in the medium in question. Therefore, an equation for the absolute index of refraction of a given material is[15]

$$n = \frac{c}{v} = \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}} \tag{4.2}$$

This does not take into consideration the index of refraction's dependence on the wavelength of light that is being transmitted. An analysis of this wavelength dependence is performed in section 4.3.

**4.2. Reflection and Transmission.** In concentrator photovoltaics, we are interested in what percentage of the available power in the incoming solar radiation can be focused onto the cell to be converted into electricity. This value is defined as the transmittance, and its counterpart is the reflectance. Together, these two components make up 100% of the power in the electromagnetic wave. The transmittance and reflectance are relatively easy to define. The following derivations have been extracted from Eugene Hecht's textbook entitled *Optics*.[15] For an incoming wave of intensity $I_i$ striking an area A at an angle $\theta_i$, the reflectance can be written as

$$R = \frac{I_r A\cos(\theta_r)}{I_i A\cos(\theta_i)} = \frac{I_r\cos(\theta_r)}{I_i\cos(\theta_i)} \tag{4.3}$$

We can write the transmittance as

$$T = \frac{I_t \mathrm{A}\cos(\theta_t)}{I_i \mathrm{A}\cos(\theta_i)} = \frac{I_t\cos(\theta_t)}{I_i\cos(\theta_i)} \tag{4.4}$$

The Law of Reflection says that the angle of incidence is equal to the angle of reflection for an incoming wave incident on any surface. Therefore, we can simplify $R$ to be

$$R = \frac{I_r}{I_i} \tag{4.5}$$

However, the same is not true for the transmittance. Snell's Law provides a relation between $\theta_i$ and $\theta_t$, but does not remove the cosine term thus does not further simply the expression. Assuming zero absorbance, the sum of the reflected wave and the transmitted wave must equal 100% of the incoming wave, as dictated by conservation of energy. That is

$$R + T = 1 \tag{4.6}$$

In addition, the intensity of light can be defined as $I = (E^2 v n \epsilon)/2$, where $E$ is the magnitude of the electric field. For our purposes, it is useful to replace intensity with this equation. For the reflectance, both the incoming wave and reflected wave exist in the same medium, delivering $v_r = v_i$ and $\epsilon_r = \epsilon_i$. Using these relationships, we find that

$$R = \left(\frac{E_{0r}}{E_{0i}}\right)^2 \tag{4.7}$$

In a similar fashion, $v$, $\epsilon$, and $n$ eventually cancel out, giving us

$$T = \left(\frac{E_{0t}}{E_{0i}}\right)^2 \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right) \tag{4.8}$$

Now, let us define the amplitude reflection coefficient as the ratio of the electric field amplitudes of the reflected wave to the incident wave,

$$r = \frac{E_{0r}}{E_{0i}} \tag{4.9}$$

Similarly, we can define the amplitude transmission coefficient as the ratio of the electric field amplitudes of the transmitted wave to the incident wave, yielding

$$t = \frac{E_{0t}}{E_{0i}} \tag{4.10}$$

Applying these new definitions to Eq. (4.7) and Eq. (4.8), we can easily write

$$R = r^2 \tag{4.11}$$

$$T = t^2 \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right) \tag{4.12}$$

Next, we need the Fresnel Equations, which define the parallel and perpendicular amplitude reflection and transmission coefficients. They are as follows:

$$r_\perp = \frac{n_i \cos\theta_i - n_t \cos\theta_t}{n_i \cos\theta_i + n_t \cos\theta_t} \tag{4.13}$$

$$r_{\parallel} = \frac{n_t \cos\theta_i - n_i \cos\theta_t}{n_i \cos\theta_t + n_t \cos\theta_i} \tag{4.14}$$

$$t_{\perp} = \frac{2n_i \cos\theta_i}{n_i \cos\theta_i + n_t \cos\theta_t} \tag{4.15}$$

$$t_{\parallel} = \frac{2n_i \cos\theta_i}{n_i \cos\theta_t + n_t \cos\theta_i} \tag{4.16}$$

In the Fresnel Equations, a $\perp$ subscript indicates a wave that is perpendicular to the plane of incidence, and the $\parallel$ subscript indicates the wave is parallel to the plane of incidence. Similarly to Eq. (4.11) and Eq. (4.12), we can define the perpendicular and parallel reflectance and transmittance using the reflectance and transmittance coefficients as

$$R_{\perp} = {r_{\perp}}^2 \tag{4.17}$$

$$R_{\parallel} = {r_{\parallel}}^2 \tag{4.18}$$

$$T_{\perp} = {t_{\perp}}^2 \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right) \tag{4.19}$$

$$T_{\parallel} = {t_{\parallel}}^2 \left(\frac{n_t \cos\theta_t}{n_i \cos\theta_i}\right) \tag{4.20}$$

The intensity of light that is parallel to and perpendicular to the plane of incidence make up the total intensity of the wave. Therefore, we can write

$$I_r = I_{r\parallel} + I_{r\perp} \tag{4.21}$$

Extending this to Eq. (4.5), we have

$$R = \frac{I_{r\parallel} + I_{r\perp}}{I_i} \tag{4.22}$$

If the incoming light is unpolarized, then by definition

$$I_{i\parallel} = I_{i\perp} = \frac{I_i}{2} \tag{4.23}$$

Applying Eq. (4.5) to the perpendicular and parallel cases yields

$$R_{\perp} = \frac{I_{r\perp}}{I_{i\perp}} \tag{4.24}$$

$$R_{\parallel} = \frac{I_{r\parallel}}{I_{i\parallel}} \tag{4.25}$$

After rearranging to obtain expressions for $I_{r\perp}$ and $I_{r\parallel}$, we have

$$R_{\perp} I_{i\perp} = I_{r\perp} \tag{4.26}$$

$$R_{\parallel} I_{i\parallel} = I_{r\parallel} \tag{4.27}$$

Using the relationship defined in Eq. (4.23), we have

$$R_{\perp}\left(\frac{I_i}{2}\right) = I_{r\perp} \tag{4.28}$$

$$R_{\parallel}\left(\frac{I_i}{2}\right) = I_{r\parallel} \tag{4.29}$$

Finally, we are able to return to Eq. (4.22), making use of Eq. (4.28) and Eq. (4.29), yielding

$$R = \frac{R_{\parallel}\left(\frac{I_i}{2}\right) + R_{\perp}\left(\frac{I_i}{2}\right)}{I_i} \tag{4.30}$$

Therefore, after cancelling $I_i$, our final equation for reflectance is

$$R = \frac{1}{2}(R_{\perp} + R_{\parallel}) \tag{4.31}$$

This relationship can also be applied to the transmittance, yielding

$$T = \frac{1}{2}(T_{\perp} + T_{\parallel}) \tag{4.32}$$

Using the equation for transmittance, we can effectively calculate the percentage of incoming radiation that is transmitted to the source. We consider this an optical loss that occurs when focusing the light onto the multi-junction solar cell. It is important to note that reflectance and transmittance depend on four variables: the index of refraction of the incoming wave's medium and the transmitted wave's medium, and the angle of reflection and transmittance of each wave. Therefore, the material used governs the values for transmittance and reflectance. To illustrate this, consider the case in which the incident wave is normal to the interface of the 1st and 2nd medium. Evidently, this would cause $\theta_i = 0$ and $\theta_t = 0$. In this case, the Fresnel Equations become

$$r_{\parallel} = r_{\perp} = \frac{n_i - n_t}{n_i + n_t} \tag{4.33}$$

$$t_{\parallel} = t_{\perp} = \frac{2n_i}{n_i + n_t} \tag{4.34}$$

Plugging these simplified equations and $\theta_i = 0$ and $\theta_t = 0$ into Eq. (4.17) through Eq. (4.20) yields

$$R_{\parallel} = R_{\perp} = \left(\frac{n_i - n_t}{n_i + n_t}\right)^2 \tag{4.35}$$

$$T_{\parallel} = T_{\perp} = \left(\frac{2n_i}{n_i + n_t}\right)^2 \left(\frac{n_t}{n_i}\right) \tag{4.36}$$

Now, after simplifying and plugging into Eq. (4.31) and Eq. (4.32), we have

$$R = \left(\frac{n_i - n_t}{n_i + n_t}\right)^2 \tag{4.37}$$

$$T = \frac{4n_i n_t}{(n_i + n_t)^2} \tag{4.38}$$

To show how the material used affects the transmitted wave, consider two materials, diamond and glass, which have indices of refraction of $n_d = 2.4$ and $n_g = 1.5$. Since the index of refraction of air, the initial medium of the wave, is approximately one, the transmittance of light through diamond is $T_d = 0.83$, while in glass it is $T_g = 0.96$. It is equivalent to say that diamond reflects 17% of the wave, while glass only reflects 4%. This is one reason not to cover our solar cells with diamonds (ignoring the huge expense!). More importantly, it is clear that a smaller index of refraction is desirable in terms of transmitting the highest fraction of the wave to the solar cell.

**4.3. Wavelength Dependence of the Index of Refraction.** As shown above, the material plays a crucial role in determining the percentage of light transmitted. The variation in material incorporates several layers of complexity, so at this point we will use a very specific function for the index of refraction for only the material $SiO_2$. This very specific dispersion equation is[16]

$$n(\lambda) = \sqrt{\frac{0.696\lambda^2}{\lambda^2 - 0.068^2} + \frac{0.408\lambda^2}{\lambda^2 - 0.116^2} + \frac{0.897\lambda^2}{\lambda^2 - 9.896^2} + 1} \tag{4.39}$$

Recall that in this derivation, we are using the AM 1.5 spectrum to represent the Sun's radiation. This is not a function, and rather a data table of wavelengths and corresponding intensities of each wavelength. Therefore, we must numerically integrate the spectrum using the index of refraction to find the transmitted power. Assuming normal incidence, we can use Eq. (4.38) to determine the transmittance of each wavelength. This will be squared, since light must enter the material and then leave the material, which creates two interfaces at which light can be reflected. We can calculate the intensity of light transmitted by multiplying each of these transmittance values by the intensities of the corresponding wavelengths, which is essentially a Riemann sum. After finding the sum of these intensities, and comparing that sum to the total intensity incident on the interface, we can determine the transmitted power. As an example of this, an excerpt of the Excel document used to calculate the following values is included below.

| | D | E | F | G | H | I | J | K | L |
|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | |
| 2 | Direct+circumsolar W*m$^{-2}$*nm$^{-1}$ | Wavelgth in micro m | Index of Refr. | % transmitted | Using 96% Transmitted | Actual Transmittance | 96% Total (W) | Actual Total (W) | Actual Efficiency |
| 3 | 2.5361E-26 | 0.28 | 1.494163661 | 0.96080047 | 7.8720E-02 | 7.8786E-02 | 1.3019E+03 | 1.3091E+03 | 0.9653 |
| 4 | 1.0917E-24 | 0.2805 | 1.493984545 | 0.96082327 | 9.5040E-02 | 9.5122E-02 | | | |
| 5 | 6.1253E-24 | 0.281 | 1.493806561 | 0.96084593 | 1.4400E-01 | 1.4413E-01 | | | |
| 6 | 2.7479E-22 | 0.2815 | 1.493629701 | 0.96086844 | 2.0352E-01 | 2.0370E-01 | | | |
| 7 | 2.8346E-21 | 0.282 | 1.493453952 | 0.9608908 | 2.5632E-01 | 2.5656E-01 | | K - J (W) | |
| 8 | 1.3271E-20 | 0.2825 | 1.493279307 | 0.96091302 | 2.9088E-01 | 2.9116E-01 | | 7.1817E+00 | |
| 9 | 6.7646E-20 | 0.283 | 1.493105755 | 0.9609351 | 3.1200E-01 | 3.1230E-01 | | | |
| 10 | 1.4614E-19 | 0.2835 | 1.492933287 | 0.96095704 | 3.1008E-01 | 3.1039E-01 | | | |
| 11 | 4.9838E-18 | 0.284 | 1.492761892 | 0.96097883 | 2.8704E-01 | 2.8733E-01 | | | |
| 12 | 2.1624E-17 | 0.2845 | 1.492591563 | 0.96100049 | 2.4023E-01 | 2.4048E-01 | | | |
| 13 | 8.9998E-17 | 0.285 | 1.492422289 | 0.96102201 | 1.6885E-01 | 1.6903E-01 | | | |
| 14 | 6.4424E-16 | 0.2855 | 1.492254062 | 0.9610434 | 1.4880E-01 | 1.4896E-01 | | | |
| 15 | 2.3503E-15 | 0.286 | 1.492086872 | 0.96106465 | 2.3232E-01 | 2.3258E-01 | | | |
| 16 | 1.8458E-14 | 0.2865 | 1.49192071 | 0.96108577 | 3.1968E-01 | 3.2004E-01 | | | |
| 17 | 7.2547E-14 | 0.287 | 1.491755569 | 0.96110675 | 3.4752E-01 | 3.4792E-01 | | | |
| 18 | 3.6618E-13 | 0.2875 | 1.491591437 | 0.9611276 | 3.2544E-01 | 3.2582E-01 | | | |
| 19 | 2.8061E-12 | 0.288 | 1.491428309 | 0.96114833 | 2.9856E-01 | 2.9892E-01 | | | |
| 20 | 9.0651E-12 | 0.2885 | 1.491266174 | 0.96116892 | 3.1200E-01 | 3.1238E-01 | | | |
| 21 | 3.4978E-11 | 0.289 | 1.491105024 | 0.96118939 | 3.7632E-01 | 3.7679E-01 | | | |
| 22 | 1.5368E-10 | 0.2895 | 1.490944851 | 0.96120973 | 4.5984E-01 | 4.6042E-01 | | | |
| 23 | 5.1454E-10 | 0.29 | 1.490785646 | 0.96122994 | 5.4048E-01 | 5.4117E-01 | | | |
| 24 | 1.3303E-09 | 0.2905 | 1.490627402 | 0.96125003 | 5.8176E-01 | 5.8252E-01 | | | |
| 25 | 3.8965E-09 | 0.291 | 1.49047011 | 0.96126999 | 5.9328E-01 | 5.9406E-01 | | | |
| 26 | 1.4425E-08 | 0.2915 | 1.490313762 | 0.96128984 | 5.7408E-01 | 5.7485E-01 | | | |
| 27 | 4.0789E-08 | 0.292 | 1.490158351 | 0.96130956 | 5.4432E-01 | 5.4506E-01 | | | |
| 28 | 7.0414E-08 | 0.2925 | 1.490003869 | 0.96132916 | 5.0784E-01 | 5.0854E-01 | | | |
| 29 | 1.5760E-07 | 0.293 | 1.489850307 | 0.96134864 | 5.1648E-01 | 5.1721E-01 | | | |
| 30 | 4.7095E-07 | 0.2935 | 1.489697658 | 0.96136801 | 5.2704E-01 | 5.2779E-01 | | | |
| 31 | 9.4558E-07 | 0.294 | 1.489545915 | 0.96138725 | 5.1168E-01 | 5.1242E-01 | | | |
| 32 | 1.5965E-06 | 0.2945 | 1.489395069 | 0.96140638 | 4.9920E-01 | 4.9993E-01 | | | |
| 33 | 3.2246E-06 | 0.295 | 1.489245115 | 0.9614254 | 5.0592E-01 | 5.0667E-01 | | | |
| 34 | 8.0206E-06 | 0.2955 | 1.489096044 | 0.9614443 | 5.3664E-01 | 5.3745E-01 | | | |
| 35 | 1.4737E-05 | 0.296 | 1.488947848 | 0.96146308 | 5.5008E-01 | 5.5092E-01 | | | |
| 36 | 2.3312E-05 | 0.2965 | 1.488800522 | 0.96148176 | 5.0016E-01 | 5.0093E-01 | | | |
| 37 | 3.3187E-05 | 0.297 | 1.488654057 | 0.96150032 | 4.5888E-01 | 4.5960E-01 | | | |

Chart1 SMARTS2 +

Table 1. This Excel document was used to perform various numerical calculations and integrals. The data in the far left column details the AM 1.5 spectrum in increments of half a nanometer.

As shown in the far right column, which was calculated by dividing transmitted power by incident power, the actual transmitted power is 96.53% per interface when accounting for the effect of the wavelength dependence of the index of refraction. Squaring this, accounting for two interfaces, yields an efficiency of 93.18%. Typically, the value used for the index of refraction for $SiO_2$ is 1.5, which yields 96%, and 92.16% for two interfaces. Therefore, an extra theoretical 1.02% additional power transmitted is obtained by numerically integrating the AM1.5 spectrum as opposed to simply taking the index of refraction to be constant. This is a small difference, but it increases efficiency in two ways. Most obviously, more power strikes the cell, and therefore more photons are available for hole-electron pair production at lower energies. A more subtle effect is a slightly higher concentration factor, as a result of the greater photon flux, which also increases the overall efficiency of the cell through $W$. In this way, a derivation that does not account for the functional nature of the index of refraction in CPV may be underestimating the efficiency by a small amount.

# CHAPTER 5

## Analysis of CPV Viability

It would seem that the results obtained in this paper are insignificant. At the end of the day, the technology that we have developed hasn't been improved in the slightest amount by these calculations. Overall, these small improvements that are possible in the calculation of theoretical efficiency maximums appear to be minor points. I would argue, however, that they are in fact extremely valuable. In the same way as discussed in Chapter 2, these theoretical efficiency limits can be used as a guideline for energy researchers. Interestingly, problems occur on both ends of theoretical inaccuracies.

As discussed at the beginning of Chapter 2, overestimating can cause researchers to ignore small gains that in reality are valuable. Underestimating, on the other hand, can be just as dangerous as well. Imagine a researcher has just discovered new tactic with a p-n junction. The new tactic allows an efficiency of 29% for a silicon cell. Because this is so incredibly close to the detailed balance limit at 30% as defined by Shockley and Queisser, they may conclude that their calculations were invalid. Using the results found in this paper, however, an efficiency of 29% is far enough away from 33.65% to be plausible. In this way, verifying the true limit imposed by p-n junctions in the AM 1.5 spectrum is a very important task to undertake, and failing to do so could potentially cause a breakthrough to be cast aside as an impossibility in the field of photovoltaics. In addition, we can use the results from each of these derivations to reach conclusions regarding the applications of each form of solar electricity generation.

**5.1. Theoretical Maximum Efficiency.** Efficiency varies across materials. For this reason, this analysis examines the most common semiconductors used in photovoltaics in order to reach relevant conclusions. In Chapter 2, we examined the detailed balance limit, which has published the maximum efficiency for a silicon single junction cell to be 30%. In Chapter 3, we found that for a GaInP/GaAs/GaInAsP/GaInAs multi-junction cell under 1000x concentration, it was 56.3%. To put these values in perspective, it is instructive to compare them to the maximum experimental efficiencies obtained. For a single junction silicon cell, this is 25.3%[17], and for a multi-junction cell under concentration, this is 46.0%[17]. Therefore, the maximum potential for improvement, in terms of the efficiencies, is 18.6% and 22.4% respectively. This result slightly favors concentrator photovoltaics. Considering that interest in concentrator photovoltaics has only recently increased, it is likely that the efficiency will rise quickly at first and then begin to plateau. On the other hand, silicon single junction cells have been widely used for many years, and significant research has been conducted to improve their efficiency. It is unlikely that those efficiencies will be increased as rapidly as CPV efficiencies. In addition, the four band gap energies chosen for the multi-junction concentrator cell were not necessarily the best possible combination. The maximum efficiency relies heavily on the band gap energy, which implies the optimization of the band gap energies to maximize efficiency is possible. Therefore, there are two ways in which concentrator photovoltaics see an advantage in maximum theoretical efficiencies. As shown above, the potential for improvement using common semiconductors favors concentrator photovoltaics. In addition, there is also a strong possibility of discovering better combinations of semiconductors to increase the maximum theoretical efficiency.

Although other single junction semiconductors exist which have higher theoretical efficiencies, they have already been thoroughly researched and have seen their efficiencies rise to near the maximum theoretical efficiency. An example of this is GaAs, with a band gap at 1.42 eV. In the Shockley-Queisser derivation, this value for efficiency is 33%. Experimental efficiencies for GaAs cells have reached 28.8%[11]. Here, the potential for improvement is still only 14.6%, which indicates it is approaching the maximum realistically achievable value. Improving this type of cell further would likely be a difficult and expensive research process.

**5.2. Efficiency on a Land Use Basis.** Using only the results obtained thus far, it is not feasible to determine the efficiency on a land use basis, because the various designs possible create too many combinations to analyze. Instead, it is more instructive to examine the data that is published currently and draw conclusions by comparing the land use footprint.

| Technology | Direct Area | | Total Area | |
|---|---|---|---|---|
| | Capacity-weighted average land use (acres/MWac) | Generation-weighted average land use (acres/GWh/yr) | Capacity-weighted average land use (acres/MWac) | Generation-weighted average land use (acres/GWh/yr) |
| **Small PV (>1 MW, <20 MW)** | 5.9 | 3.1 | 8.3 | 4.1 |
| Fixed | 5.5 | 3.2 | 7.6 | 4.4 |
| 1-axis | 6.3 | 2.9 | 8.7 | 3.8 |
| 2-axis flat panel | 9.4 | 4.1 | 13 | 5.5 |
| 2-axis CPV | 6.9 | 2.3 | 9.1 | 3.1 |
| **Large PV (>20 MW)** | 7.2 | 3.1 | 7.9 | 3.4 |
| Fixed | 5.8 | 2.8 | 7.5 | 3.7 |
| 1-axis | 9.0 | 3.5 | 8.3 | 3.3 |
| 2-axis CPV | 6.1 | 2.0 | 8.1 | 2.8 |
| **CSP** | 7.7 | 2.7 | 10 | 3.5 |
| Parabolic trough | 6.2 | 2.5 | 9.5 | 3.9 |
| Tower | 8.9 | 2.8 | 10 | 3.2 |
| Dish Stirling | 2.8 | 1.5 | 10 | 5.3 |
| Linear Fresnel | 2.0 | 1.7 | 4.7 | 4.0 |

Table 2.[18] Land use requirements for various methods of collecting solar energy. The concentrator photovoltaics that have been discussed in this paper correspond to 2-axis CPV in the table.

Typically, silicon solar panels are fixed axis. From Table 2, the Small PV fixed axis reaches 4.4 acres/GWh/yr, while Small PV 2-axis CPV only requires 3.1 acres/GWh/yr. In Large PV, this is 3.7 acres/GWh/yr compared to only 2.8 acres/GWh/yr. From these results, it is clear that concentrator photovoltaics are more efficient on a land use basis in terms of their electricity generation.

Another triumph of concentrator photovoltaics becomes relevant when considering the environmental impact of covering the Earth with solar cells. The effect of converting the solar energy to electricity is minimal. Most of this energy will be converted to heat eventually, and therefore has little effect on the Earth.

More importantly, the cell intercepts radiation that would normally be available to plants. In this way, building a utility-size solar farm in an open field would result in the death of the vegetation, which needs sunlight to grow. At this point, concentrator photovoltaics display a property that makes them the better choice for this situation. In order for the module to safely rotate without striking another module, it is best to space them out significantly. Although this increases total land use, it allows a substantial amount of sunlight to strike the ground. In this way, it becomes possible for the land to be used for agricultural purposes, as well as for solar energy production. This vastly increases the potential for the applications of concentrator photovoltaics. Farmers, who own large plots of open and sunny land, would be inclined to lease several small portions of their farm to build each tower. Their crops could still grow, and they would receive additional income from the utilities, which would pay for the use of their land.

Overall, concentrator photovoltaics are clearly the best choice in terms of reducing land usage and minimizing environmental impacts. This stems from their ability to more effectively convert solar energy into electricity on the basis of solar panel area, as well as on a land use basis. In addition, the spacing between concentrator photovoltaic modules creates the potential for agriculture to continue beneath the towers, allowing a hybrid use of the land. These factors highlight a major advantage that concentrator photovoltaics possess over silicon single junction photovoltaics.

**5.3. Levelized Cost of Electricity (LCOE).** The LCOE is a measurement thats integrates nearly all of these factors. The results obtained in sections 5.1 and 5.2 are taken into account by determining the price at which the electricity can be sold at and examining the cost of the total land use for each application. Other factors that are crucial to the calculation of this number are the actual cost of the solar modules, the cost of maintenance and cleaning, and the engineering costs.

As mentioned in Chapter 1, the LCOE for concentrator photovoltaics ranges from $0.08/kWh to $0.15/ kWh. For single junction photovoltaics, the value is much lower, ranging from $0.03/kWh to $0.12/kWh[19]. Although somewhat obvious, the reason that these values must be represented as a range is due to the unique circumstances of each installation site. Depending on the landscape and the availability of sunlight, the costs may vary. It is possible for values to be found outside of these ranges, but on average it is likely that the LCOE will fall between these values.

As discussed in Chapter 1, this is the primary reason that concentrator photovoltaics have not performed well in the last fifteen years, while solar energy has seen a tremendous boom in production (see Fig. 1). Unless government subsidies are introduced, it is unlikely that concentrator photovoltaics will be used on a large scale in the United States. At the end of the day, the utility companies determine where our energy comes from, and the LCOE plays a huge role in determining their position on various forms of energy. However, it is likely that concentrator photovoltaics will become more economically attractive in the near future as companies learn to manipulate the advantages of CPV over single junction PV in their favor. Concentrator photovoltaics will continue to grow in use, but improvements are required before it can inexpensively be implemented on a large scale.

# CHAPTER 6

## Conclusion

According to this analysis, it is clear that concentrator photovoltaics have demonstrated high viability as a method of sustainable utility-scale electricity production. However, it may be advantageous to continue expanding single junction PV until the efficiency of CPV improves, and the LCOE decreases to a competitive level. Implementing concentrator photovoltaics at their current high costs may result in less total solar cell installations. This is unfortunate, because concentrator photovoltaics have demonstrated several beneficial assets.

Aside from price, concentrator photovoltaics have the most attractive attributes. CPV integrates multi-junction cells, which convert solar energy to electricity more efficiently by capturing energy that is inaccessible to single band gap p-n junctions. In addition, they are under concentration, meaning that less semiconductor material is required. This concentration, although incorporating further optical losses, also increases the efficiency by increasing *W,* the work done per photon. Moving onto the issue of land usage, concentrator photovoltaics again came out on top. They are more efficient per acre of land that is allocated for electricity production. Almost more importantly, it remains possible to use the unused land around the CPV module towers for agriculture. This minimizes the impact of installing these generators across the country on a large scale. This property of producing a low environmental footprint is key, and should be strongly considered. The replacement of our old generating facilities is a golden opportunity, and the energy systems selected should be carefully chosen in order to minimize future repercussions.

Replacing our current energy infrastructure with sustainable alternatives is an important goal for our country to have. Although price should be taken into consideration, long-term effects may not have a price tag associated with them, and should be taken into account. Renewable energy technologies are headed in the right direction, and it is important that the progress made in the last fifteen years is built upon in the future. The primary priority, of course, should be to maintain a strong economy and continue to fuel its processes with as much energy as it requires. However, as long as sustainable energy investments do not undermine our country's financial situation, they should be vigorously pursued. Concentrator photovoltaics have significant potential, and should absolutely continue to be considered for applications in many regions of the country.

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