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Meredith L. Greer
Bates College, mgreer@bates.edu

Chip Ross

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SENIOR SEMINAR: ACROSS A DEPARTMENT AND ACROSS THE YEARS

Meredith L. Greer
Department of Mathematics
Bates College
Lewiston, ME 04240, USA
mgreer@bates.edu

Chip Ross
Department of Mathematics
Bates College
Lewiston, ME 04240, USA
sross@bates.edu

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Abstract: The LaTeX class file `primus.cls` is designed to include the information needed to electronically submit a manuscript for publication in *PRIMUS*. We offer details and related suggestions on its use and submitting manuscripts.

Keywords: senior seminar, capstone, assessment

1 INTRODUCTION

We write, on behalf of our entire department, to describe the senior seminar at Bates College: its origin, its structure, its successes and difficulties, and its formal assessment by the College. As the quotes above show, nearly every senior — in fact, an estimated 97–98% of seniors — writes a thesis. Mathematics is one of the rare departments that offers another alternative. We believe that this combination of offerings attracts more majors, allows a meaningful capstone for every senior, and helps the faculty in our department to balance the demands of providing a culminating experience for each senior mathematics major.

In Section 2 of this article, we set the stage at Bates and compare the two types of capstone options for Mathematics majors. Section 3 outlines the goals and structure of our senior seminar, with specific illustrations from a recent seminar detailed in Section 4. In Section 5, we offer tips to others who may be offering a seminar-style capstone, based on the experiences in our department. Section 6 describes three types of assessment that have provided insight about our seminar.

2 CAPSTONE IN CONTEXT

Bates College is a small liberal arts college located in Lewiston, Maine. Each semester we have about 1750 students on campus. Our Mathematics department has seven full-time faculty members who each teach five

courses per year. We also have a part-time faculty member who teaches two courses per year. Besides classroom teaching, all faculty at Bates are expected to advise theses or otherwise contribute to senior capstone experiences.

For Mathematics majors, "capstone" means either a thesis or a senior seminar. The thesis is a one- or two-semester project, usually completed one-on-one with an advisor from our department, and culminating with a public presentation of results and a written thesis document. Thesis writers receive one or two course credits, based on the number of semesters spent writing. In nearly all disciplines at Bates, thesis is the only available capstone. The Mathematics department, however, also offers the option of a one-semester senior seminar, worth one course credit. This seminar has the feel of a graduate-level seminar, organized by a professor, but with most class meetings led by students. Oral presentation and communication are the skills most emphasized in seminar; writing is also very important.

In their junior year, mathematics majors must decide whether they prefer thesis or seminar. A student preferring to take the seminar need only register for it. A student preferring to write a thesis must decide on a topic and write a proposal before the end of the junior year. In either case, it is important that students actively choose, so that they are maximally engaged and enthused. For the seminar, the instructor chooses the topic, and the department publicizes the following year's seminar topics and professors at least two months before the thesis proposal deadline. In all, typically half or more of our rising seniors select the seminar. In the most recent few years, our number of senior majors per year has increased from the low teens to high teens, while the number of thesis writers has not increased. To accommodate the additional demand in our senior seminars, we have increased the number of seminars per year from one to two. Table 1 lists titles of all senior seminars offered since their inception in the 1998-1999 academic year.

3 THE GOALS AND STRUCTURE OF THE BATES SENIOR SEMINAR

The department faculty have agreed on several common objectives for the senior seminar. Although the department has no formal list of rules for the structure of individual seminars, a fairly standard format has proven effective. Instructors are free to modify this format and we encourage each other to experiment with other ways to meet the objectives.

Two fundamental goals are to have students learn an area of mathematics beyond the standard course work, and lead the classes with their own presentations of this new material. Often the instructor runs the class for the first two weeks or so, laying down a common background

in the subject by giving lectures and perhaps some exercises. Then the students take over, and their presentations make up the rest of the semester. Class-time is given over to these presentations. Students are expected to read, study and work on their presentations, and meet with the instructor for help, outside of class.

Another primary common objective is to have the class members work in small groups of two to three. Early on, the instructor gives each group its own first reading assignment, and the group members are responsible for learning the material and putting together a presentation of it for the rest of the class. Because all the seminar participants have a common background in the subject, these presentations should be understandable to the whole class. The readings the instructor chooses may be from upper-level texts or journal articles. There are usually three or four such readings per group over the semester. Groups typically work their readings into solo presentations which together cover the group's assigned topic in a coherent well-connected way.

Good mathematical writing is a very important common goal of our seminars. For example, students may be asked to write up presentations in the style of a short mathematics paper. These papers often require one or more revisions as the instructor helps the students by constructively criticizing their written work.

Before their senior year, our majors have had LaTeX instruction in multiple courses. Continued development of LaTeX skills is an important part of the capstone. The bidirectional role of LaTeX is crucial. Of course, it makes written mathematics look nice. But this cosmetic ability helps to improve the actual content and style of writing. The art of typesetting forces consideration of how much detail to give, whether development is well-planned, whether examples are clear, and so on. Students additionally learn BEAMER, a LaTeX-based method of preparing slides that aids in giving quality mathematical presentations.

Undergirding all these goals is the enjoyment of mathematics as we learn it, talk about it with others, and marvel at the creativity and far-reaching aspects of our discipline.

Additional goals are at the instructor's discretion. For example, in a seminar that makes intense use of computer graphics, an instructor may stress the importance of good design of meaningful figures with appropriate labels, descriptions, and connections to the article text. Another instructor may want to encourage students to seek out readings in journals on their own, gaining skills in searching for the right content at an appropriate level of complexity. A seminar may involve heavy use of some package such as *Mathematica* or Maple, and a fluency of a certain level may be a goal. Some instructors include a focus on history of mathematics. The production of a coherent course document, containing works by all seminar participants, may be a desired tangible outcome.

4 CASE STUDY: A LOOK AT A RECENT SEMINAR

In this section, we'll take a more detailed look at an actual seminar as conducted by this paper's second author. This seminar was held during the winter 2011 semester. Twelve students signed up for the seminar, a number considerably larger than our target of six to eight students, and this caused some concern since more students meant less time for presentations by each student, more time between talks for individual students, and less time for the instructor to devote to each student.

Front cover of the course document, which is a compilation of all seminar participants' papers and presentation. A copy of this document is available at [link to be provided].

The students themselves were wonderful to work with. Since our college is small, the students already knew one another well and had been in previous classes together in various combinations. They divided themselves into four study groups, and each group worked together very well. The students obviously enjoyed watching each other learn and present the material.

The chosen topic, Chaotic Dynamical Systems, lends itself beautifully to a seminar such as this. After some background, there are many directions one can follow. Students may look at chaos in real-world systems in areas such as physics or biology, or they can consider the topic

for its mathematical content alone. There is room for theory in different topics, computer explorations, and study of both real and complex systems.

The instructor led the class for three weeks. The first day began with a discussion by the students about what characterizes a good talk and what elements are undesirable. Then the instructor was challenged to demonstrate these ideals, setting an example for the students to follow later, as we began building a common background in the subject. Students learned fundamental definitions of terms like *dynamical system, orbit of a point, fixed* and *periodic points*. As a class, we proved the attracting fixed point theorem, which is a great application of the mean value theorem. We discussed searching for points of period n and categorizing them as attracting, repelling, or indifferent. We did this using analytic, numerical, and graphical techniques to find solutions of the equation $f^n(x) = x$ and to evaluate required derivatives. (Here f^n means the composition of f with itself n times). These background discussions and take-home exercises culminated in a whole-class project: Students found the points of various periods for different members of a one-parameter family of functions and categorized them as attracting, repelling, or indifferent. The class combined their findings and graphed the results on a single large sheet to produce a bifurcation diagram.

At this time, the instructor assigned each of the four groups its first topic for eventual presentation to the whole class. The groups were responsible for reading and learning the material outside of class, with instructor help as necessary. Each group decided how to break the topic into three parts and which members would discuss them. The first speaker in each group was responsible for beginning his or her presentation with an abstract of the whole group's presentation. This first series of talks was delivered "lecture-style", with the speakers presenting at the blackboard and using live computer demonstrations or saved pictures as appropriate. Here is a list of the first-series topics assigned by group number:

1) Development of saddle-node and period-doubling bifurcations in families such as $f_c(x) = x^2 + c$.
2) Orbit *vs* bifurcation diagrams.
3) Chaos.
4) Complex-valued dynamical systems; introduction to Julia sets.

In particular, Group 2 divided their topic among presenters A, B, and C as follows:

A) Seeing both attracting and repelling cycles on a bifurcation diagram for $f_c(x) = x^2 + c$.
B) Algorithms for plotting orbit and bifurcation diagrams and some features of each diagram.

C) Using a bifurcation diagram to illustrate the Sarkovskii ordering; showing the density of periodic points in $[-2, 2]$ for $f(x) = x^2 - 2$

The seminar met three times per week at 80 minutes per meeting. Classes at Bates meet anywhere from 160 to 240 minutes per week, using more than the minimum when the instructor deems it pedagogically beneficial. Knowing in advance the seminar would have a lot of students, the instructor scheduled this one for the maximum, ideally allowing for two 40 minute presentations per day. During the fourth week the instructor devoted class time to helping students prepare for their talks and holding discussions with them about their divisions of the material. Presentations began in the fifth week and spilled into the seventh week. The instructor purposely did not attempt to have all of one group finish before the next began. This kept all groups engaged in their work, allowed for flexibility, and challenged us all by having several topics under consideration in any given week.

Students were required to take notes on all talks except those given by their own group members. The students were responsible for turning in these notes accompanied by a short summary of each talk.

After presenting, the students wrote up their own talks formally in LaTeX. This required group collaboration as the members of each group were instructed that the three papers were to flow together as one. Additionally, although hand-drawn and live computer demonstrations were acceptable during the talks, the written papers required professional-looking figures, and to the extent possible, original examples (*i.e.*, different in some ways from those in the source). While the students worked on these papers, they also were given their second round of topics to begin learning and preparing. The students found this to be a pretty heavy load, and no talks were ready for a week. Some of the material itself was quite difficult and required instructor involvement in mastering it.

The talks went very well. All students were prepared, and some of them went overboard as they got quite into the subject. The written work was equally good. The instructor critiqued each paper and met individually with all the students to discuss revisions. In general, although wording needed tweaking or figures needed modification or typos showed up or equations got "misTeXed", the results were the students' own.

The second round of talks required BEAMER presentations. There were two challenges now: The material was more difficult, and the students had to think carefully about what definitions, results, and examples to put on each slide. Live computer demos were allowed, and students could use the blackboard sparingly as appropriate.

To set the tone for what would be expected in such BEAMER presentations, the instructor gave the first one, a review of bifurcation diagrams. To varying degrees, each group's second-round topic was an

extension of the first one:

1) Preperiodic points and the bifurcation diagram of $f_c(x) = x^2 + c$.
2) The "period-three" theorem. (Very loosely: If there's a point of period 3, there's one of *any* period n).
3) P and Q curves for $x^2 + c$. (Q-curves are the shadowy curves that weave through a standard orbit diagram).
4) Continued explorations of properties of Julia sets.

Note-taking in the 21st century: After a long night of working on a theorem with the instructor, "mobile devices" make it easy to take notes.

In addition to meeting with the instructor for discussion and mild revision of their BEAMER slides, students had to prepare one final round of talks. These would be shorter as the end of the semester was rapidly approaching. The final topics were:

1) Preperiodic points and bifurcation diagrams of some simple rational functions.
2) Algorithms for locating repelling points of various periods on Julia sets.
3) P and Q curves for other some functions.
4) The Mandelbrot set, some interesting properties, and its connection to Julia sets.

Students discovered some genuinely new mathematics during the seminar. In the second round of presentations, one Group 1 student found that a certain circle appears implicitly in a bifurcation diagram of preperiodic points. The instructor is very curious to study this! And during the last round, a student in Group 3 determined numerically that

slopes of Q and P curves for the family $c\,e^{\sin x}$ interact according to the same formulas as they do for the family $x^2 + c$. This was a surprise. Now to prove it!

> Something new! Why do we see a circle in the right hand picture of locations of pre-periodic points?

The final product of the seminar was the creation of a comprehensive course document. This collection consisted of a formal write-up by the instructor of his introductory lectures, followed by all the students' first-round papers and then the BEAMER talks. Putting this together was more of a challenge than the instructor anticipated. There were many technical details, such as giving the talks and slides a uniform appearance. But the resulting full color 130-page document was worth the effort, and early in the summer, copies were mailed to all the seminar participants.

5 CONCERNS AND STRATEGIES

One of the biggest issues we've faced is how to keep the whole class engaged during presentations. While the common background is there, the material in any given presentation can still be difficult for classmates. We have developed various ways to help the listeners stay focused. In some seminars, the students who just presented are asked to take careful notes on the next group's presentation and "LaTeX-up" these notes. In

other seminars, students are asked to take notes on what their peers present and pass in these (hand-written) notes along with their own short summaries of the important ideas. In still other seminars, students are asked to provide a peer review of their classmates' presentations, perhaps on a standardized form. The review might include the presenters' clarity, organization, correctness, and facility with the material. And finally, some seminars experiment with having the presenters design homework problems to be done by the rest of the class.

In any course that emphasizes group work, group dynamics are always at work, and making sure each group member plays a full role is a specific concern. One aid is to have the first presenter in a group give an abstract of the group's entire presentation — this means the first member doesn't just do the "easy stuff". Rotating the order of presenters in a group from one of their topics to the next is a good idea. Although it is very time-consuming, having a group give a practice presentation to just the instructor (before the formal in-class talk) can assure work will even out. Occasionally it helps to give a group member his or her own individual topic-within-a-topic. This "ownership" can be an encouraging, positive experience for a student, especially when the student has come up with his or her own question about something in the material. Some seminar instructors have a policy that requires students to rate their group members' efforts, either confidentially or not.

We emphasize that no one seminar can do it all: there are many good ways to achieve these goals and handle issues that arise. At Bates we do not expect that as instructors we will consider all goals as equally important, nor try all the different techniques. What is important to us is ongoing encouragement of one another's experiments, constant discussions with our peers about how things are going, and offering advice and constructive feedback. All of this effort is designed to help our students develop more fully as mathematicians.

6 ASSESSMENT

Assessment for our senior seminar has come in multiple forms. After each course at Bates, students complete online evaluations. Professors can use this feedback to improve the seminar the next time they teach it. These evaluations are confidential, however, and only the course's professor can see the comments students write. Because of this confidentiality, indeed unavailability, we do not include end-of-semester evaluations in this paper.

In summer 2009, Bates graduates from 1990–2008 with majors in the sciences and mathematics were surveyed. Our second assessment, this survey was sponsored by the Bates Dean of Faculty's office and funded by the Howard Hughes Medical Institute. Goals focused on how alumni

went on to use their math and science majors: Did they stay in these fields? Did they pursue further degrees in math or the sciences? Did Bates give them the background they needed for their math and science goals? While this assessment provided many valuable insights, none of the questions focused directly on senior capstone experiences. A few math majors referred to their thesis, and only one referred, indirectly, to the senior seminar: that student, who graduated more than ten years ago, felt that in not writing a thesis, he or she had taken the easy way out. This thinking concerns us, to be sure. However, this student was at Bates when the senior seminar first became available as an option for math majors. We knew that in early days, students wondered if seminar was a less rigorous or less valuable option. We have worked hard to change the culture so students equally value both thesis and seminar. We emphasize that the seminar and thesis have some goals in common, and some independent goals, and that both experiences are equally challenging and worthwhile. We also organize a roundtable discussion in which current seniors discuss their capstone choices with sophomores and juniors, answering questions from their own perspectives.

The third assessment focused directly on senior capstones and provides additional, and more recent, information. This project was organized by the Bates Office of Institutional Research and Assessment (OIRA) with assistance from the Dean of Faculty's office. For each major at Bates, these two offices facilitate a series of three meetings of the faculty in that department or interdisciplinary program. Subsequently, a staff member from OIRA interviews outgoing seniors about their capstone experiences. The mathematics department held our three meetings in winter and spring, 2010. We began to formalize the goals for our major and the ways our capstones contributed to students achieving those goals. We then read three documents as a group: two theses and one document produced by a senior seminar course. We used these common readings to refine our list of goals.

Our goals list for majors consisted of five overarching categories: Develop Mathematical Maturity, Logical Thinking, Communicate Effectively in All Forms, Intellectual Development, and The Basics. Within these categories, we listed more detailed goals, such as intellectual independence, developing problem solving skills, speaking about math before a variety of audiences, risking failure for the opportunity to succeed, and enjoying mathematics.

We next discussed the extent to which we had already been achieving our goals and how to publicize them to current majors, both before they select their capstone and while they are completing it. This is an ongoing project. Some of the best advice for publicizing our goals arose from the next part of this assessment: interviews with outgoing seniors.

Five seniors sat down with OIRA for interviews. All were from the

same year. Given the small sample of students interviewed, the results are in some ways limited. However, the students provided many insights into the thought processes of some of our recent graduates, particularly regarding their senior capstone decisions.

One single member of OIRA conducted all the interviews. In her summary of the results, the interviewer reported that all the students felt well prepared for their capstone experience, be it thesis or seminar. She found that students selected their capstone based on anticipated senior course load, and in particular, whether they were writing a thesis in another major. Another factor was student awareness of the steps needed to write a thesis. In some cases, students had been studying abroad junior year, and they did not always feel well enough aware of thesis proposal deadlines; as a result, they felt they had defaulted into seminar, and were therefore less happy about their capstone.

Specific to the seminar, the OIRA interviews show that students valued their senior seminar topic, the presentation skills they developed, and the opportunity to work on their mathematical writing skills. During the interviews, students read the departmental list of goals, and above all, they highlighted the communication skills they had learned from senior seminar. The goals chart resonated with students, some of whom were pleasantly surprised to see the ways their professors thought about the major.

Regarding the choice between seminar and thesis, one student provided a key insight: we should make the goals and a timeline available to students earlier in their mathematical careers, preferably in a course taken by all majors. The student identified Introduction to Abstraction, our gateway proofs course. All majors must take this course at Bates; they cannot transfer it from elsewhere. This was a flashbulb moment for departmental faculty: we wonder why we hadn't thought of it ourselves. Starting this year, we will hand out the goals and a timeline for majors and take time to discuss these documents during Introduction to Abstraction, which students usually take in their first or second year. We plan to further publicize the goals via our website and departmental bulletin boards and redouble our efforts to contact students studying abroad in the junior year.

A few quotes best represent the variety and individuality of responses. The first describes advantages of the seminar for a senior with a double major.

> "I think it's probably good to do a thesis. But if [as a double major] you've already done a thesis, which I think a fair amount of math majors [do], I wouldn't [do a math thesis]. The senior seminar is a different type of skill, where you do a lot more group work, and you do a lot more presentations, and stuff like that. . . .

[I]f you can do both, it's even better."

The second quote addresses the cons and pros of sharing departmental goals with our majors.

> "I don't know if students would necessarily read [the goals table] over and, you know, think about it too much.... But...it's cool thinking about it after. I feel like I've done most of the things on that list, so seeing it now makes me feel more accomplished than I did like even at the beginning of this meeting, which is cool."

We conclude with a quote that summarizes how we hope all students feel about senior seminar.

> "Just have fun and enjoy yourself. You're in a class with a lot of other math majors; you probably know them. It can be a pretty fun class. You can work through some interesting problems and you can get to really hang out in a mathematical setting and then you hang out in like the 'you're chilling with your friends' type of way. You can do that and you can do math, so it's a pretty good opportunity for that."

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BIOGRAPHICAL SKETCHES[1]

A. Author received her education at a liberal arts college in New Jersey and has been interested in applications of mathematics throughout her entire career. She seeks to engage her students in a search for relating mathematics to everything. She likes to think about all these ideas while mountain climbing and walking her dog—sometimes at the same time.

B. Author developed an interest in art while participating in an NSF sponsored workshop and has gone on to exhibit his work in New York galleries. He is also a brain surgeon on the weekends and enjoys walking his dog—never at the same time!

[1]In this unnumbered section offer up a short, paragraph form, biographical sketch of each author. This, of course, should be stripped (see Section **??**) from the anonymous version of the file.