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Interdisciplinarity and Inclusivity: Natural Partners in Supporting Students

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Abstract: *Mathematics Across the Sciences* is an applications-based course designed to strengthen students' mathematical skills in preparation for calculus or for a major in the sciences. Its development relied heavily on input from faculty members in several science departments. This article describes the course; many of the scientific applications taught; pedagogical strategies; and scholarship on inclusion, equity, and diversity in education. These descriptions make clear that the goals of inclusion and academic excellence are intertwined, and an interdisciplinary approach can help each to improve.

Keywords: math education, science, applications, diversity, inclusion

1 INTRODUCTION

> *Do you have difficulty making connections between the math you learn in math class and the math you are asked to use in your science class? How does that formula for the equation of a line relate to this set of data you just collected in physics, chemistry, biology, geology, or environmental studies? Do you need a good solid review of ratios, linear relationships, exponents, logarithms, graphs, and more? If you want to be able to apply your mathematical knowledge in other contexts and strengthen your core mathematical skills, this course is for you.*

This course is *Mathematics Across the Sciences*, created and taught at Bates College. Its motivations, content, and teaching methods draw from (at least) three major sources: (i) input of faculty from every science department on our campus, (ii) a multidisciplinary faculty working group on diversity and inclusion, and (iii) nationwide math education conversations.

Mathematical content focuses on number sense, metric units, graphs, and functions. Each math topic is introduced and motivated with at least two applications from different areas of science. Assignments and assessments include traditional homework and exams alongside a variety of methods designed to get students excited about, and invested in, the current state of mathematics and science. Class time, course topics, and out-of-class assignments are designed to encourage a diverse set of students to succeed in this course as well as when they later proceed to more advanced mathematics and science courses. As will be shown, all the sources (i), (ii), and (iii) contributed to these aspects of the course.

This paper describes the creation process for the course, with Sections 2, 3, and 4 elaborating on sources (i), (ii), and (iii) listed above. Section 5 concludes with both results from planned assessment and serendipitously acquired feedback on the course. For further details: Appendix A dives into the many applications explored in this course, while Appendix B provides insight into in-class and out-of-class teaching techniques for reaching students who started as hesitant, even fearful, about their mathematical abilities.

2 CONTRIBUTIONS OF SCIENCE FACULTY

The creation of *Mathematics Across the Sciences* relied on meetings with all of the science departments at Bates College. The two primary questions at meetings were "What examples, from courses *you* teach, could be included in a math course as examples of how math concepts are important?" and "What mathematical topics do you want your students to learn better?"

Initially, each department hosted a meeting, and faculty brainstormed enthusiastically. Eventually, trends surfaced. For example, Chemistry, Physics, and Mathematics faculty wanted their students to have stronger algebra and calculus skills. Biology, Geology, and Environmental Studies faculty noted a surprising amount of student trouble with graphs and ratios.

Later, each department suggested one to two of their members for ongoing discussions and an exchange of resources. These meetings led to further refinements of mathematical topics and scientific examples to include in the course.

Faculty typically mentioned specific applications in their courses first, such as Biology's expectation that students be able to work with dilutions in a laboratory setting. After compiling ideas from multiple departments, we organized applications into mathematical categories (such as "proportions" for the dilutions example) as follows:

- unit conversion;
- the metric system;
- number sense;
- proportions;
- basic algebra skills;
- graphs;
- functions;
- lines;
- exponential growth and decay;
- logarithms;
- basic trigonometry.

Using this set of categories, faculty members sought a wide range of applications for each. Examples emerged from all the departments listed earlier. After a serendipitous conversation, the Psychology department got involved with yet more examples. Professors contributed class notes, extra copies of textbooks, tips about online resources, and a great deal of time and expertise to the project. For those interested in using some of this paper's ideas in classes: some key articles and websites appear in the bibliography. Other resources include textbooks for introductory courses in biology, chemistry, astronomy, physics, geology, and environmental studies, all of which were valuable for explaining concepts and producing examples for student practice.

After the selection of content, presentations to many constituencies—department chairs, science faculty, the mathematics department—kept interested colleagues up to date on concrete plans for the course. Appendix A summarizes the mathematical topics and scientific applications shown in these presentations.

3 DIVERSITY AND INCLUSION

During the development of *Mathematics Across the Sciences*, Bates College facilitated a multidisciplinary faculty working group titled "Crossing Borders: Engaging Difference, Educating for Change." The results were powerful. One catalyst for the group was the Making Excellence Inclusive project of the Association of American Colleges and Universities (AAC&U) [8], which emphasizes that the concept of diversity includes a wide range of characteristics: "Individual differences (e.g., personality, learning styles, and life experiences) and group/social differences (e.g., race/ethnicity, class, gender, sexual orientation, country of origin, and ability as well as cultural, political, religious, or other affiliations)". The working group's members met, read articles, discussed approaches that could be used in different styles of classroom teaching, and created modules for immediate use in our courses.

Some articles we read emphasized student perspectives. The longitudinal study in [4] reports on students' positive and negative experiences, in academic and social settings. Ours was one of the focal campuses of the article, making these findings especially relevant to our student body. Students of color quoted in [10] make clear their passion for academic learning, alongside the difficulties they sometimes have fully accessing channels of learning. They seek a variety of classroom activities, attention from faculty and TAs inside and outside the classroom, and the opportunity to make mistakes, rather than have instructors rush to show them answers. One student says, "Professors... should try to make classes enjoyable for everyone. There needs to be more interaction with students and students should be allowed to ask more questions." Another tells us, "If too much information is thrown out at once, which is usually the case, there is no chance for internalization. You spend a lot of time trying to copy what is on the board... and not really listening to what they are saying." All these comments connect back to the student-centered teaching goals mentioned in Section 4, emphasizing that such pedagogy is beneficial for all kinds of students, not only underrepresented students.

We next explored specific interactions between professors and students, guided in part by the complex and nuanced set of studies in [3]. This book chapter describes studies of the effects of critical feedback given by white professors to black students, and by male science professors to female students. When these professors give *unbuffered* feedback, that is, only comments indicating work is incorrect or not of high enough caliber, students typically react by guessing the professor is biased. As a result, the students put far less effort than their peers into revision and improvement. By contrast, when students receive *wise* feedback, pairing critical comments with a stated expectation of high standards and the professor's confidence that the student can achieve those high standards, the students work more intensely at revision, typically achieving notably greater improvement than their peers. Other studies in [3] report the tendency of many professors to criticize underrepresented groups *less* and offer them *more* praise, in comparison with other students. Interviews indicate that this behavior arises from professors' concerns about unfairly or overly criticizing students from underrepresented groups, but [3] emphasizes research showing this lack of critical feedback can communicate low standards and otherwise impede student progress. The chapter culminates by emphasizing the need for trust: each student needs to know the professor cares. But how can a professor achieve this? Besides crafting *wise* feedback on assignments, professors can be available to students outside of class time, and can take an interest in students' lives outside of academics. Of great importance, professors and students benefit from “framing ability as malleable rather than fixed.”

One last important resource comes directly from the AAC&U. Their writings on the Making Excellence Inclusive project [9] define diversity to emphasize how people *engage with differences*, not just by difference itself. They describe a framework for colleges and universities to follow, including a detailed chart to show how campuses can function in ways that encourage excellence, diversity, equity, and inclusion. For example: whereas in a traditional model, students show excellence by having high GPAs, in an inclusive campus, students also show resilience “pursuing academic endeavors and in the face of academic and personal challenges”. Faculty in a traditional model show excellence via good teaching evaluations; in an inclusive campus, faculty also are “able to teach broadly within their own discipline and help students make connections to other disciplines”. The chart has many more instances of “traditional” and “inclusive” hallmarks, categorized by “Students”, “Faculty Members”, “Administrators and Staff Members”, and “The Curriculum”, and is an invaluable resource for departments and campuses pursuing meaningful change.

Along with reading articles, our working group created pedagogical plans that we agreed to start using within the next semester. Because the group’s members drew from a wide array of disciplines, we could readily compare approaches between, say, a course that often uses problem sets and a course that relies primarily on reading and discussing articles. Each faculty member had the dual opportunity to plan activities with other faculty teaching similar styles of course, yet also hear from everyone else, permitting a cross-fertilization of pedagogical approaches across our disciplines.

This working group played a huge role in developing the pedagogical strategies used in *Mathematics Across the Sciences*. Details about these strategies appear in Appendix B.

4 NATIONWIDE THOUGHTS ON MATH EDUCATION

The motivations driving the development of *Mathematics Across the Sciences* are not all new. Dating to the early 2000s, for example, the Mathematical Association of America's Curriculum Foundations Project [7] tells us that "[s]tudents do not see the connections between mathematics and their chosen disciplines; instead, they leave mathematics courses with a set of skills that they are unable to apply in non-routine settings and whose importance to their future careers is not appreciated."

This Curriculum Foundations Project report goes on to describe a set of recommendations, including the following, all of which were central to the thought process for development of *Mathematics Across the Sciences*:

- "Use models from the partner disciplines: students need to see mathematics in context."
- "Require students to explain mathematical concepts and logical arguments in words. Require them to explain the meaning—the hows and whys—of their results."
- "Strive for depth over breadth."
- "Use a variety of teaching techniques.... [E]ncourage the use of active learning, including in-class problem solving opportunities, class and group discussions, collaborative group work, and out-of-class projects."

The same report encourages faculty members to "pay attention to units, scaling, and dimensional analysis" and "[i]mprove interdisciplinary cooperation", which our science faculty discussed directly as goals when we met to construct the set of course topics and examples.

Meanwhile, the past several years have seen a blossoming of the Inquiry-Based Learning (IBL) community among mathematicians. Articles, blog posts, and a host of conference sessions and workshops all engage faculty sharing IBL ideas and resources. What constitutes IBL can vary from professor to professor and from course to course, but descriptions often express the student-centered nature of IBL, and the shift of the professor's role from "dispensing knowledge to... facilitating learning" [15]. IBL classrooms may focus on active learning, problem solving, student discovery of mathematical ideas, and communication [5].

At the same time, stories abound of math anxiety in the general public.[1] In particular, *Mathematics Across the Sciences* was expected to, and does, draw in students who are unconvinced of their mathematical ability.

Concern about mathematical preparedness extends beyond the mathematics education and STEM communities. It reaches all the way to the President of the United States. In February 2012, the President's Council of Advisors on Science and Technology presented the report *Engage to Excel: Producing one million additional college graduates with degrees in science, technology, engineering, and mathematics.* While some aspects of the report ruffled feathers among many mathematicians—notably the call for "college mathematics teaching and curricula developed and taught by faculty from mathematics-intensive disciplines other than mathematics, including physics, engineering, and computer science" [6]—the rest of the report tracks well with goals of the Curriculum Foundations Project and IBL practitioners. Notably, the report includes a table of active learning techniques, each of which is supported by peer-reviewed scientific study. The techniques include small-group discussions, use of clickers, problem-based learning, problem sets done in groups, and computer simulations and games. These overlap heavily with the Curriculum Foundations Project's suggestions, quoted above, as well as with techniques used in IBL classrooms and in the student-centered approaches of many mathematics educators, and they are all part of the pedagogical approach to *Mathematics Across the Sciences*.

This course was planned with all of the above considerations in mind. Class meetings thus intentionally included a variety of hands-on techniques, paired with context-driven activities outside of classroom hours, as part of encouraging students to take ownership of their mathematical

[1] It seems likely that this article's audience will be well acquainted with such stories. An online search for "math anxiety" returns a large number of them, with new articles appearing frequently.

learning.

5 ASSESSMENT AND UPDATES

Mathematics Across the Sciences has been offered three times. Student feedback from in-class discussions and end-of-semester evaluations provides a glimpse at the strengths and shortcomings of the course. Each offering of the course benefits from changes based on the students in previous semesters: their successes, their struggles, and their suggestions.

From the start, many evaluation comments were very positive. Students especially enjoyed the applications to sciences, seeing that mathematics is alive and in use all around them. Many students appreciated the mental math component and the opportunity to strengthen their number skills. One particularly significant comment follows: "THIS is the kind of class that should be duplicated for people seeking their [quantitative] requirements. Those of us in the humanities, social sciences, or the arts should not have to struggle through a tortuous and irrelevant math or science class in order to get a C or low B. This class proceeded at a perfect pace for the lowest common denominator, and the emphasis was on LEARNING USEFUL SKILLS (imagine that?)."

This same comment, however, points to one of the two criticisms some other students had about the course: that the material was too elementary for them. Most of these commenters said they liked the applications but wanted to see more advanced math concepts; some added that they had taken AP Calculus in high school. This pointed to a necessary update: formally restricting the course to students who had not taken too much advanced mathematics already, rather than relying on the course number and college catalog description to signal the course level. The current solution is to permit students to take *Mathematics Across the Sciences* if they already have credit for Calculus I, but not if they have credit for any math course at a higher level than Calculus I. The reason is that some students take Calculus I or AP Calculus AB and subsequently realize they need to review earlier concepts before they can successfully take more advanced mathematics. Most students who take courses beyond Calculus I, however, are by then too advanced for the mathematical content of *Mathematics Across the Sciences.* (The instructor can always make an exception if an individual student states a strong case for taking the course. In practice, no one has requested to do so.) Restricting course access to students with less mathematical background has been a great help.

The second criticism from the first offering of this course was that students had too many Bring It In and Attending Outside Events assignments: five of each! (Descriptions of each are in Appendix B.) The current requirements of four and two, respectively, respect that each assignment takes a great deal of time, and give students more chance to focus on and enjoy each assignment.

Through the subsequent offerings of the course, students have generally enjoyed the content and structure, but this is not always unanimous. Below are some of the concerns consistently presented by a few students each semester. These are offered in the spirit of solidarity with educators seeking to always improve teaching and to excite students about learning. Teaching a math course using non-standard methods frequently leads to such concerns, and while each of these has been outweighed by far more student evaluators commenting that they liked these teaching approaches, it is important to listen to every student and continue exploring ways to draw in each one of them.

- Not everyone likes the Bring It In and/or Attending Outside Events assignments. Some students like one but not the other; some think the assignments are too difficult; some say they should be for extra credit only.
- Some students dislike group work or discovering mathematics, asking instead that the instructor explain everything first, thoroughly, before asking students to practice the concepts.
- A few students feel there are too many different assignments and too many pieces to their grades.
- Some students request different topics or more depth in some topics, such as statistics, algebra, or trigonometry.

Beyond evaluations administered by the instructor or the college, some students have offered other feedback in incidental meetings on-campus. The semester after she took *Mathematics Across the Sciences*, one student got in contact to say she was seeking her teaching certification and believed that this course had prepared her exceptionally well for the math portion of her upcoming Praxis test. Multiple students in the second and third offering of the course said they selected *Mathematics Across the Sciences* because they were interested in teaching and had heard that this course showed exciting ways of teaching mathematics. (Our campus offers an education program, but has not consistently provided courses focused on science or math teaching.) Several students went on to take Calculus I and stated that they felt much more comfortable in Calculus I than they would have without *Mathematics Across the Sciences*. Several additional students pursued science degrees and reported that *Mathematics Across the Sciences* increased their quantitative and mathematical confidence in ways that made it possible for them to continue their science education.

Designing for inclusivity played a central role and had far-reaching effects. In-class and out-of-class activities were based on research into how to engage all students, yet many were new to the author's teaching skill set. Teaching *Mathematics Across the Sciences* was thus daunting, yet exciting. The experience gained, especially from the visible successes of new teaching approaches, generated confidence and ideas for how to use such activities elsewhere. Infusing similar teaching approaches into other courses has led to improved communication with students, enhanced student enthusiasm about course content, and even an opportunity to create a course with the very different diversity of mixing advanced math majors with students having no previous college mathematics experience. A focus on puzzle-solving, investigating applications, and working in teams made that course accessible yet challenging for all. It is worth noting that many mathematics professors were students ourselves long before faculty typically used student-centered teaching approaches, so our teaching careers started without role models for such techniques. Updating our teaching may therefore require an incremental approach as well as plenty of discussion with fellow educators about how best to move forward. Seeing the resulting delight in students, especially when they are from a wide variety of backgrounds, is one of the best rewards for making these changes.

To conclude, this course was designed to prepare students for more advanced math and science; to connect mathematics to several scientific disciplines; to offer support on the path into math and science for students from diverse educational backgrounds; and to illuminate for students the central role of mathematics in science, news, and current events. The course has largely been successful, and it will continue to evolve to reach as many students as possible and to stay current with scientific research.

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BIOGRAPHICAL SKETCH

Meredith L. Greer enjoys applications of mathematics, with particular research interest in mathematical epidemiology and ecology. Her students and colleagues continue to introduce her to new connections to math. Recent projects have involved softball pitching, roller coasters, community structure in HIV models, and the intersection of mathematics and Berlin history.

A APPENDIX: APPLICATIONS FROM ACROSS THE SCIENCES

Table 1 summarizes most of the content of *Mathematics Across the Sciences* and was presented to campus colleagues for feedback before the first offering of the course.

Math Topic & Details	**Examples**
Lines	
Writing equations	Hooke's Law [C, P]
Interpreting slope	Relating humerus to height [B]
Interpreting y-intercept	Density [P]
Proportions	Dilutions [B]
	Human memory scanning [Ψ]
	Distance = (Rate) * (Time) [P]
	Fish weight $\propto$ eggs laid [B, E]
	Island age vs. distance from hot spot [G]
Exponents and Exponential Graphs	
Positive exponents	Aquaculture population = f(pond size) [E]
Negative exponents	Initial reaction rates [C]
Exponent = 0	Allometric scaling: $y = ax^b$ [B]
Fractions in exponents	Tumor growth [B]
Exponential growth	Doubling time [P]
Exponential decay	Half life & radiometric dating [G, P]
Graphing	Elimination of drug in the body [B]
Creating decay equations	Atmospheric pressure vs. altitude [P]
Logarithms	
Graphing	Richter & moment magnitude scales [G]
Connect to exponentials	Decibels [P]
Different bases	Memory $\approx$ (constant) − (logarithm) [Ψ]
Functions-of-functions	Hick's Law [Ψ]
Log-of-exponential $\approx$ line	pH [C]
Trigonometry	
Triangle trigonometry	Trajectories [P]
Unit circle	Temperature of a city across years [E]
Sine curve patterns	Sound waves [C, P]
Amplitude, vertical shift	Solar irradiance [E, P]
Hands-on understanding	

Table 1. Math topics and science applications. Examples arose from the following disciplines: [B] biology, [C] chemistry, [E] environmental studies, [G] geology, [P] physics, [Ψ] psychology.

Below are the details on a selection of these topics.

A.1 Lines

Hooke's Law, $F = kx$, tells us the force F required to displace a spring having spring constant k by a distance x. This topic provides practice graphing and interpreting lines, practice with algebra (given two of the three of F, k, and x, solve for the third), and practice with units. International System (called SI) units for force and displacement are newtons and meters, respectively; students may be given information in, say, centimeters, and need to convert to meters.

Relating humerus length to height provides compelling use of data in large classes. Students each measure their humerus from elbow to shoulder, and measure their total height. All pairs (*humerus height, total height*) are graphed in Microsoft Excel, which can draw a best-fit line through the data points. Students then use the line to respond to general interpretive questions, such as "For every one-inch increase in humerus length, how much taller is the average person?" as well as forensics-based questions, such as "What is the likely height of a person with a 13 inch humerus?" Forensic anthropologists typically use different graphs for men and women, which can lead to additional class discussion and different interpretive questions.

Human memory for some tasks appears to behave linearly. A 1966 study of memory [13] combined pioneering cognitive psychology with mathematical analysis. Subjects in this study memorized a small number of symbols. They were then shown a symbol that may or may not be one they memorized. As quickly as possible, subjects were to indicate "yes" (the symbol was on the memorized list) or "no" (the symbol was not on the list). Subjects who had memorized more symbols took longer to respond, and the increase in time fit a linear graph. The graph provides ample room for interpretation. Follow-up work [14] relates differing mental impairments to measurable change in y-intercept and/or slope, providing further fodder for class discussion on graph interpretation.

The ages of the mountains in the Hawaii-Emperor seamount chain

NO concentration	H_2 concentration	Initial reaction rate
0.1 M	0.1 M	1.23×10^{-3} M/s
0.1 M	0.2 M	2.46×10^{-3} M/s
0.2 M	0.1 M	4.92×10^{-3} M/s

Table 2. Data from reaction experiments with nitric oxide and hydrogen

are approximately proportional to their distances from the volcanically active Hawaii site. Images and graphs appear in [12].

A.2 Exponents and Exponential Graphs

For the aquaculture example, students consider circular ponds and compare pond radii and areas. The class explores the relationship between increasing radius length and increasing area, noticing that doubling the radius corresponds to quadrupling the area. This topic connects to the formula $A = \pi r^2$.

We consider chemical reactions such as

$$2\mathrm{NO} + 2\mathrm{H_2} \rightarrow \mathrm{N_2} + 2\mathrm{H_2O},$$

comparing initial concentrations of each chemical and the initial rate of the reaction as in Table 2. Concentrations are in molar units (M), indicating moles per liter. In this experiment, doubling the starting concentration of hydrogen (H_2) doubles the initial reaction rate. However, doubling the initial concentration of nitric oxide (NO) quadruples the initial reaction rate! Using the rate equation for this reaction,

$$(\text{Initial rate}) = (\text{constant}) \times [\mathrm{NO}]^k \times [\mathrm{H_2}]^m,$$

where [ ] indicates concentration, we deduce that $k = 2$ and $m = 1$. Initial reaction rates simultaneously highlight the importance of exponents and provide valuable algebra practice.

The growth of some tumors and bacteria populations can be modeled as exponential. Students consider equidistant time steps as doubling periods, such as time steps 0, 1, 2, 3, and corresponding bacteria populations 5, 10, 20, 40, creating charts and graphs to represent exponential behavior. Once the doubling concept is clear, students consider percentage growth in a more general context.

Radiometric dating uses the idea of exponential decay. Just as exponential growth considers doubling times, exponential decay considers halving times, also known as half-lives. Radiocarbon dating is particularly well known: carbon-14 has a half-life of 5730 years and can be used to date bones and other organism remains. Students graph remaining quantities of carbon-14 across several halving times and build visual evidence for why radiocarbon dating cannot be used for items older than approximately 60,000 years. Many other types of radiometric dating exist, from estimating the ages of ancient rocks via rubidium-strontium dating, utilizing a half-life of 48.8 billion years, to determining the years when sediment formed its layers at the bottom of a lake via the 22.3-year half-life of lead-210.

A.3 Logarithms

Students learn about the Richter scale for measuring earthquakes, and about the moment magnitude scale that officially replaced the Richter scale in 2002. The class starts by graphing the quantity of TNT required to produce a similar force as integer Richter scale values, with TNT quantity on the vertical axis and Richter scale values $1.0, 2.0, \ldots, 10.0$ on the horizontal axis. The TNT values, easily found via an online search, follow an exponential pattern quickly identified by students. However, vast variation in the scale of TNT values makes the graph difficult to draw, thus motivating the need for logarithms.

Loudness is measured via decibels, named after Alexander Graham Bell: a *Bel* is a unit of sound. The formula for loudness L is

$$L = 10 \log_{10}(I/I_0)$$

where I is intensity of a sound wave and I_0 is the minimum intensity detectable by the human ear.

Cognitive psychologists measure how humans forget things and find that forgetting can be modeled logarithmically. For example,

$$y = 81 - 12\log_3(t+1)$$

might approximate average exam scores (y) for a group of students taking an exam t months after studying for it.

Hick's Law describes the relationship between decision-making time and the number of possible choices. Reaction time T is given by

$$T = b \cdot \log_2(n+1)$$

where there are n choices, all of equal probability, and where b is a constant used to fit a given data set. Hick's work in the 1950s quantified observations, dating to the mid-1800s, of increased reaction time as the number of choices increases.

A.4 Trigonometry

We introduce trigonometry via trajectories. As one example, consider a softball pitch. If the ball is released at a measurable positive angle from the horizontal, and its total initial velocity is known, can we determine the horizontal component of its velocity? As a follow-up: in college softball, the distance from pitching rubber to home plate is 43 feet. Once we know initial horizontal velocity, and if we make the simplifying assumption that horizontal velocity does not change during this short flight, can we determine how much time passes from when the ball leaves the pitcher's hand till when it crosses home plate?[2]

Solar irradiance, a measure of the sun's brightness, approximates a sine curve when graphed across several years.

[2]As a fascinating side note, the distance from pitcher's mound to home plate is longer in baseball, but pitches are generally faster, so that the typical time the ball is in flight from pitcher to home plate is almost identical between baseball and softball at the college level.

The weather in a given city can generate many a sine graph: average high temperature on each date, average low temperature on each date, and minutes of daylight per 24-hour period are all examples. Gathering these data[3] across several years produces a graph that is, or closely resembles, a sine wave. Students interpret graph features such as amplitude and vertical shift and learn how to write sine function formulas using these features. As a sample discussion, our class compares total daily light in Lewiston, Maine with total daily light in Miami, Florida: on December 21, Miami has 100 minutes more light than Lewiston, but on June 21, Lewiston has 100 minutes more light than Miami! A fall semester course reaches the topic of trigonometry during some of the shortest days of the year, making this daily light conversation tremendously compelling.

B APPENDIX: TEACHING TECHNIQUES

The structure of assignments, grading, and in-class time all play a crucial role in student engagement. The creation of *Mathematics Across the Sciences* therefore focused not just on science applications, but also on how the course would be taught. Here we discuss the details, connecting course structure to the goals of diversity, inclusion, equity, engagement, and student preparation for further training in mathematics and science.

First, a few generalities: Maximum class size is forty students. Course material draws from a wide variety of sources, so students do not have a textbook for this class. (They openly praise the saved money!) Students learn to participate actively during class time. Only half the course grade comes from exams; the other half comes from attendance, participation, homework, and writing assignments.

Next, the specifics: in-class time involves both technologically enhanced instruction and some very low-tech time for hands-on exploration. Examples follow in sections B.1 and B.2. Outside of class, in addition to completing traditional homework exercises, students engage with mathematics and its applications and write about the results.

[3]The National Weather Service is an excellent source.

Three types of assignments are described in sections B.3, B.4, and B.5.

B.1 Technology in the Classroom

Our college's course management system serves as a home for all course-related content and assignments. A preview of each day's material appears online at least two days before the class in which that content will be discussed, benefiting students who process concepts better during class when they have had time to think about those concepts in advance. Immediately after class, all materials from that day's activities are posted, including solutions and results developed in class. Besides providing information to any students who were absent, this helps the bookkeeping of all students, which becomes particularly important in a course with no textbook.

Most class days include a PowerPoint portion with clicker questions sprinkled throughout. After each clicker question, students discuss answers in small groups and report to the full class on their clicker responses, describing which answers they selected, why, and whether small-group discussion caused them to change their response. One type of useful clicker question causes many students to respond incorrectly, yet allows most students to arrive at the correct response after small-group discussion. Another helpful style of clicker question may list several correct answer choices, with small-group discussion leading students to confirm for themselves that more than one answer is appropriate. Yet another approach is to pose two similar clicker questions, one early in the discussion of a new topic, and another after much discussion of that topic, to ascertain the progress in students' understanding. These are just a few of the ways clicker questions can be used. (For faculty members considering work with clickers or similar technology, [2] contains a treasure trove of ideas on enhancing student engagement via thoughtful clicker usage.)

The PhET website [11] has several interactive applets—games and activities—for science and math exploration, including for Hooke's Law, initial reaction rates, and radiometric dating.

While discussing the humerus topic described in Section A, the class uses Excel to graph data and draw a best-fit line. To investigate class-wide results, students must sort out when to refer to actual measurements and when to refer to the trend observed in the best-fit line. The accompanying discussions promote deep understanding about using original data versus analyzing mathematical trends.

We use videos to demonstrate some applications, such as bacteria populations doubling several times.[4] A tip to mathematics educators: applications open the doors to far more online content than mathematics-only topics, with videos engaging some students much more than other uses of class time.

Small groups start with tables of exponentially increasing values, then use calculators to compute the logarithms of those values. Different groups compute the logarithms with different bases. Each group graphs its results, which are approximately linear. As a result, students see that logarithms applied to fast-increasing data can distinguish true exponential growth (with the result being a line) from other sorts of fast growth (with results nonlinear, usually concave down).

Several of the above examples of in-class activities use technology recommended for engaging students, such as clickers, computer games, and videos [6]. Research on clickers, in particular, shows improved student performance in terms of learning and analyzing content [2]. The Curriculum Foundations Project [7] and students quoted in [10] both support these examples of active learning [7] and connect active learning to improved student success and engagement.

B.2 Hands-on Activities

Separate student groups compute (by hand!) values for 2^x, 3^x, and 10^x, where x takes on values -1, 0, 1, 2, 3, 4, 5, and 6. The class gathers results at the chalkboard. Students then compute products such as $2^1 \times 2^4$ and $3^2 \times 3^4$ by hand, comparing their results with values already on the chalkboard. After several comparisons, they develop the exponent

[4]https://www.youtube.com/watch?v=gEwzDydciWc

rule $a^m \times a^n = a^{m+n}$. They generate several other algebra rules in a similar fashion.

Students graph their computed values of 2^x, 3^x, and 10^x by hand and compare the graphs, discovering fundamental features of exponential graphs.

Starting with just a graph or just a table of data, students develop their own equations of the form $y = A \cdot 2^{(-x/B)} + C$ for exponential decay. They discover for themselves the interpretations of A (which relates to the initial y-value under consideration), B (half-life), and C (vertical offset from a horizontal asymptote at 0; the value of A depends on C). This activity also leads to valuable discussion about using different bases in exponential formulas.

Students master $x = \log_b y$ via both applications and this formula's relationship with $y = b^x$. They discover they can solve many logarithms by hand once they understand exponent rules.

Manipulatives help with trigonometry understanding. Students fit paper cut-outs of a $45° - 45° - 90°$ triangle and a $30° - 60° - 90°$ triangle onto a paper unit circle. One angle of a triangle is positioned at the center of the circle, then students compute corresponding x- and y-values on the circumference of the circle. The results determine the sine and cosine values for the angle at the circle's center.

These activities fulfill many goals. Our science faculty stressed the need for number sense and graphing ability; both sets of skills improve via frequent in-class and homework practice. The Curriculum Foundations Project calls for collaborative group work, active learning, and depth over breadth [7]. Students quoted in [10] seek a variety of in-class activities, interactivity, and time to ask questions. During these hands-on activities, there is plenty of time for students to ask questions, both of others in their small groups, and of the professor.

B.3 Bring It In

These assignments—four per semester—require students to seek articles from outside sources. Students then write a one-page paper summarizing the article, identifying its mathematical content, and explaining article insights they have gained from our in-class math and science discussions. Bonus credit goes to each student who reports on an article that no one else in the class selects.

The specific assignments are:

1. Select an article from *Discover* or *Scientific American* that uses math.
2. Identify data that can be fit with a straight line, or with a curve having an exponent (e.g. x^2 or x^{-1}). The data may come from *either* an article from an online news source, *or* a data set you collected in another course you have taken at Bates College.
3. Locate a science article involving exponential growth or exponential decay. Sources may be science magazines, professional science journals, or online news articles.
4. Locate a science article involving logarithms, trigonometry, or oscillation. Permissible sources are the same as in assignment 3.

Observations and Helpful Hints

- Students tend to find fantastic articles you may not have found on your own. This alone can be reason to assign Bring It In papers!
- It is difficult to define what constitutes an online *news* source. Discuss this in detail with your students, or you may receive assignments from wacky sources. Guidelines may include well-known news outlets and/or a clearly stated editorial process. Blog posts without editors, and articles with unidentified sources, may be examples of what not to submit.
- Similarly, consider defining an *article* as a full and original article, not an educational summary of long-known information (such as online class notes or encyclopedia entries) or a stand-alone chart, graph, or table of data, without an accompanying story.
- It helps to encourage students to check that their proposed articles fit the requirements, as they can not always judge this themselves.

Also helpful is explicitly recommending they start early in seeking articles, so they have time to ask for approval of their sources.

- The word "exponential" is persistently misused in our culture, even in scientific writings, when the growth is merely fast. Use caution and discuss thoroughly when asking students to locate truly exponential growth in outside sources.
- Students do far better searching for applications, such as decibels or the Richter scale, than for "logarithm". A similar remark applies to exponential and trigonometric applications. Therefore students start their Bring It In assignments on a particular mathematical topic *after* the class has begun exploring related applications.
- Bring It In assignments spotlight the startling frequency with which mathematical concepts are described in ways that avoid actual mathematics, even in professional science journals! A spirited discussion about math phobia, editorial decisions, and our culture's priorities may well ensue.
- Current-day scientists are a much more diverse set of people than the historically famous scientists whose ideas we usually study in class. This assignment allows students to read about scientists and mathematicians from varied backgrounds and upbringings, thus generating greater opportunity to find scientists who look like our students.

B.4 Attending Outside Events

Students must attend two math or science events on campus and write about what they saw. Their instructions are: "In approximately one page, single-spaced, summarize the content of the event you attended. Be sure to highlight any mathematical content and the role it played in scientific work. If you cannot identify any mathematical content, state this explicitly." They can submit their papers anytime during the semester, up until a stated end-of-semester deadline.

Observations and Helpful Hints

- Invited speaker talks, class poster sessions, student thesis presentations, and visiting candidate job talks are typical event options.
- Appropriate events can be found campus-wide and are designated on the course management system. This serves two purposes: advertising, and clarifying which events "count". Students are also encouraged to suggest events, but must receive approval before using an event for this assignment.
- It is important to designate events at a variety of times, so that all students can find options fitting their schedules.
- Many science talks have no mathematics at all, and that is OK for this assignment. Students do their best to identify mathematics in an event, and otherwise they write about what they saw. Experience shows that requiring two events leads almost every student to see math presented at least once.
- These events' interactivity can bring out a previously hidden side from some students. A student who seemed quiet or unsure of mathematics may ask an elegant math question of a speaker, or may draw on knowledge from other courses in a way that makes connections and shows serious academic thought. Students sometimes surprise themselves with their interest in the event topic and find themselves in a deep dialogue with a speaker or poster presenter.

B.5 Posting to our Class Forum

Our course management system has a Forum feature, permitting back-and-forth interaction between professor and students. Most weeks include a prompt in the Forum, and students respond for credit. The Forum setting is on one-on-one communication, so student responses go only to the instructor, not to other students.

Sample prompts include:

- (In Week 1) Can you name a math concept you learned in the past that you found especially interesting? If so, what was the concept?
- As we move toward Exam 1, what topics are clearest for you, and

what do you most want to practice before the exam?

- We regularly use in-class time to try math without calculators, such as drawing graphs by hand. Is this a valuable use of class time?
- Describe an everyday situation involving straight lines. What does slope mean in this context? How about y-intercept? The idea here is to identify a straight line in a non-math, probably non-science, context.
- How is the class pacing these days? Too fast, too slow, just right?
- (During registration week for the following semester) Are you thinking about more math, whether formally as a class, or seeing math in daily life after this semester? Your answer here can be open-ended: if you're taking another math class, I am interested in hearing about it if you'd like to tell me about it. Or you can talk about aspects of math you expect to continue seeing after this semester is over.
- Find an example of exponential growth or decay, logarithms, or trigonometry in everyday life. Non-math-seeming examples are ideal here! Explain your example and why you believe it represents or uses exponential growth or decay, logarithms, or trigonometry.

Observations and Helpful Hints

- The Forum is an easy and fun exercise, and it provides huge opportunity for one-on-one communication with students. You can use this interaction to encourage students to visit you in-person in your office, or to take more math in a later semester, for instance. Personal communication also helps build trust between student and professor, which [3] reminds us is crucial to helping students perform to a high standard.
- This is some of the most low-stakes writing students do all semester: students earn full credit for any reasonable response, and the tone can be very conversational. Low-stakes writing has tremendous benefits for both students and professors, as described in [1], which details many more writing ideas tailored to quantitative courses.
- The Forum generates some of the greatest insights into student

> thinking! Some students feel less comfortable speaking in class, but chat openly in the Forum. Others show deep mathematical understanding, or make startling connections across disciplines, in ways that other assignments and class time do not permit.

The teaching techniques and types of assignments described above encourage frequent and varied student interaction with mathematics and its applications. Students can personalize what they learn, for example via their selection of articles and events for course assignments. Embracing applications of mathematics leads to rich opportunities for students to feel connected to course content. Students express what they have learned in a variety of ways, so that nearly all students find a way of understanding, explaining, and enjoying mathematics and its uses.

The pedagogical approaches used in *Mathematics Across the Sciences* are shown in Sections 4 and 3 to both increase the diversity of mathematics students and to benefit *all* students. Incorporating applications provides depth of mathematical understanding, both in mathematics for its own sake and in the ability to use math in the partner disciplines. The Curriculum Foundations Project [7] pointed out that mathematicians should focus on these goals when educating students. Putting it all together, we need not choose between encouraging high standards and valuing diversity, inclusion, and equity. We need not choose between depth of mathematical knowledge and having enough time to discuss applications of math. Instead, it is natural that these all occur together.