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On a sangaku of Sugino'o Shrine (Yamagata) and Yamaguchi Kanzan's second trip

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Abstract

In the preamble of the 1818 sangaku tablet of Sugio Shrine, the proposers acknowledged the help of an unnamed teacher/master in understanding and solving certain mathematical problems. Endō Tadashi argued that this unnamed teacher could be Saitō Naonaka (1773-1844). In this paper, we examine the famous travel diary of Yamaguchi Kanzan (?-1850) especially on his second trip to the Northeast. We compare the content of Yamaguchi's diary with the three problems of Sugio's tablet. Together with the timing of Yamaguchi's travel, we conclude that Yamaguchi Kanzan was likely the unnamed master mentioned in the preface of the Sugio Shrine sangaku.

Keywords: Wasan, sangaku, Yamaguchi Kanzan
2000 MSC: 01A27

1. Introduction

Since the publication of Rothman and Fukugawa's article [4] in the *Scientific American*, works published on wasan and sangaku in western languages have significantly increased. Their book, *Sacred Mathematics* [3], further opened the door for the western readers to a glimpse of the popularity of wasan in Edo Japan. They devoted Chapter 7 to a translation of Yamaguchi's 3rd journey based upon the travel diary [11] of the famous itinerant mathematician Yamaguchi Kanzan 山口和 (? - 1850). Other works include the extensive book by Horiuchi [5] and a recent English translation of a popular book by Sakurai [7].

At the height of wasan popularity in mid to late Edo period (1603 - 1868), many itinerant mathematicians travelled throughout Japan teaching mathematics to common people and were most welcome everywhere they went. From 1817 to 1828, Yamaguchi took 6 journeys traveling to all parts of Japan

Figure 1: Sugio Shrine, Tsuruoka City, Yamagata Prefecture (1818)

and he recorded in his diary over 350 mathematical problems with over 250 such problems taken from over 85 sangaku tablets most of which have since been lost. Evidently, Yamaguchi met and discussed with local mathematicians and many students from various mathematical schools during his trips.

On the first of August in 1818, a sangaku tablet was made by a group of students from the Ōyama [大山] mathematics school and was devoted to Sugio Shrine 椙尾神社 in Tsuruoka City in Yamagata prefecture. The preamble (Figure 2) of this tablet stated that the students were given guidance by a "master recently" (頃者得師家).

Endō argued in his article [1] that this master was Saitō Naonaka 齋藤尚仲 (1773 –1844), a disciple of Aida Yasuaki 会田安明 (1747 –1817), the founder of wasan school Sai jo-ryū (最上流). In this note, we claim that this master was likely to have been Yamaguchi Kanzan.

2. The Sangaku of Sugio Shrine

The sangaku of Sugio Shrine (Figure 1, see also [10]) contains a preamble and 3 geometric problems. It was dated 1818, 1st of August (in the lunar

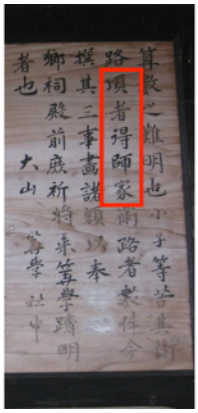

Figure 2: Preamble of Sugio Shrine Sangaku

calendar) and offered by the Ōyama mathematical school (in Ōyama district of Tsuruoka City).

Problem **1** (*yōjutsu*)

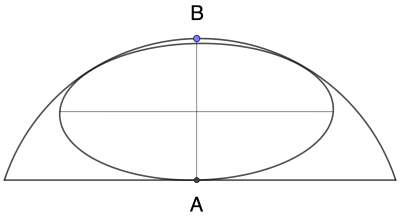

Figure 3: Problem 1 of Sugio sangaku

Consider Figure 3. Suppose the major axis of the ellipse is 6, the minor axis is 3, and the distance between A and B is 3.1. Find the diameter of the circumscribing circle.

Problem **2** (*chikusakujutsu*)

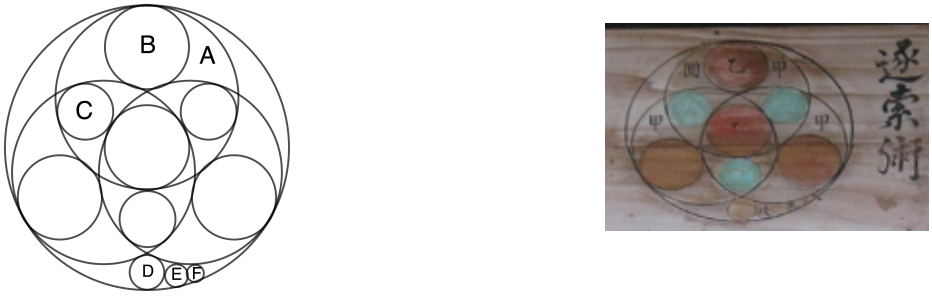

Figure 4: Problem 2 of Sugio sangaku

As shown in Figure 4. Suppose the diameter of the circumscribing circle is given, find the diameters of the corresponding circles $A, B, C, D,$ *etc..*

Problem **3** (*tetsujutsu*)

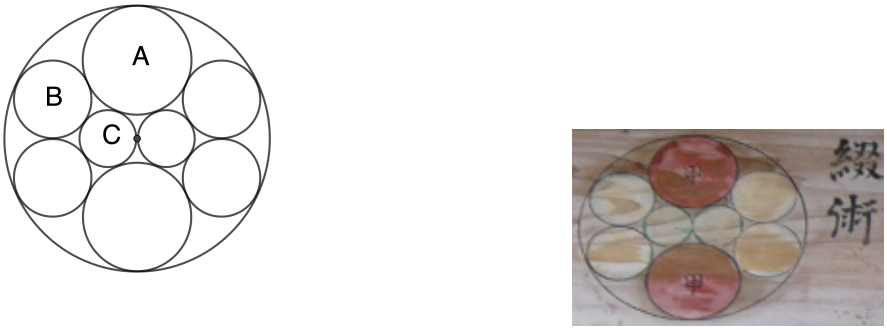

Figure 5: Problem 3 of Sugio sangaku

As shown in Figure 5 in which the circumscribing circle contains two A circles, four B circles, and two C circles. Suppose the circumscribing circle has diameter 10*, find the diameters of circles* $A, B,$ *and* C*. [Better not to use square roots.]*

In his argument that the master mentioned in this sangaku was Saitō Naonaka, Endō's [1] main argument was that problem 1 and problem 3 also appeared (almost in verbatim) in Saitō's draft notebook dated around 1822. He also pointed out that Saitō's mathematical influence in the Tohoku region

appeared to be earlier than 1822. Indeed, in his travel diary Yamaguchi recorded from an 1817 sangaku of Koshiō Shrine 古四王神社 (Akita) 16 problems posed by students of Saitō Naonaka.

3. The sangaku of Koshiō Shrine

In 1817, a sangaku containing 16 problems was devoted to Koshiō Shrine in Akita City. Unfortunately, this tablet has been lost and most probably was destroyed during the great fire of 1888, after which the current shrine was re-built. On the other hand, Yamaguchi visited Koshiō Shrine on the 1st of July in 1818 [9, p.35] [1] and recorded all 16 problems in his diary.

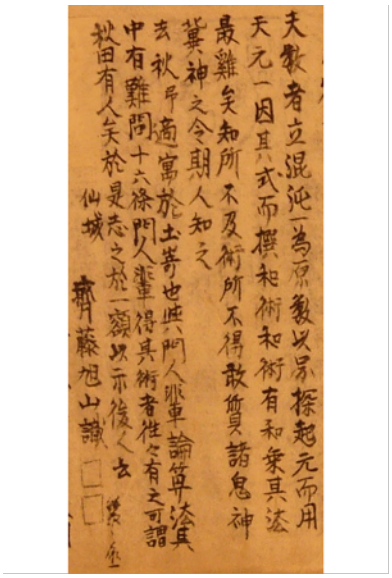

Figure 6: Preamble - Koshiō sangaku

From the preamble of this tablet (see also [2, p.42]), as recorded by Yamaguchi, Saitō Naonaka (also known as Saito Asahiyama (斎藤旭山)) wrote (second paragraph of Figure 6) “Last Autumn, while staying in Tsuchizaki, I discussed 16 difficult mathematical problems with my students. Those who solved these problems are considered notable [mathematical] representatives from Akita. Therefore this tablet is made to display their solutions for future

[1] This book does not reproduce any mathematical problems from the original diary.

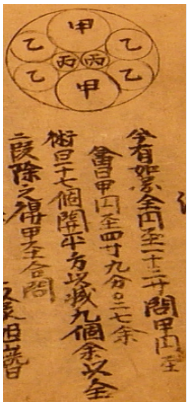

Figure 7: Prob 10 –Koshiō sangaku (same as Prob 3 –Sugio sangaku)

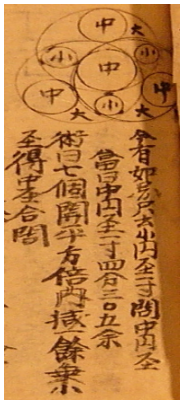

Figure 8: Prob 6 –Koshiō sangaku (similar to Prob 2 –Sugio sangaku)

generations." Moreover, of the 16 problems on this tablet, problem 10 is the same as problem 3 of Sugio sangaku (Figure 7); and problem 6 is essentially the same as problem 2 (Figure 8) of the Sugio tablet.

Since the problems of the lost Koshiō sangaku were proposed by the students of Saitō, as clearly stated in the preamble of the tablet, this confirms that Saitō was aware of problem 3 of the Sugio sangaku and that Saitō was probably the person who suggested such a problem to his students in the first place.

While Endō did not comment on problem 2 of the Sugio sangaku, it is clear that the problem is essentially the same as problem 6 of the Koshiō sangaku. The similarity between these two problems can be explained as

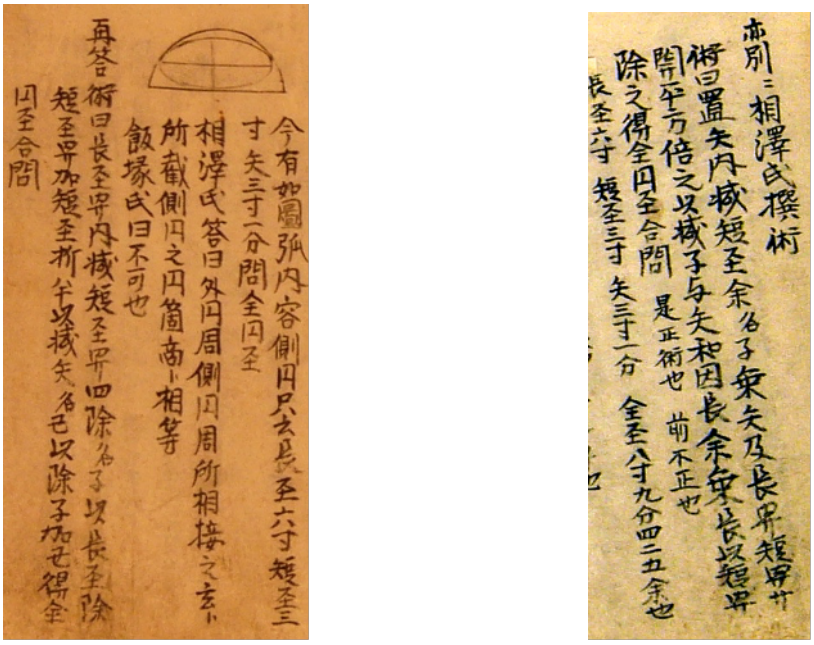

Figure 9: (left) Iizuka's problem and Aizawa's solution (right) Another Aizawa's solution with answers

follows. In Figure 8, by symmetry there is a circumscribing circle containing the three large (大) A circles, the four medium (中) B circles, and the three small (小) C circles. The additional circles D, E, F, etc. in Figure 4 will then be uniquely determined.

4. Yamaguchi's second trip to the Northeast

From October 1817 to September 1818, Yamaguchi travelled from Edo (nowadays Tokyo) to the Northeast including Akita and Yamagata prefectures. According to his diary, Yamaguchi visited Noshiro City (Akita) on the 15th of May 1818 [9, p.33]. In discussing with Iizuka (a student of Saitō) and Aizawa, he copied a problem which is (in verbatim) the same as problem 1 of the Sugio sangaku. He also recorded the solutions (answers) to the problem.

Interestingly, Yamaguchi copied in his diary an exchange (Figure 9) between Iizuka and Aizawa which led to Aizawa's second solution that he claimed to be the correct one while his previous solution was not. Furthermore, Aizawa's answer for the diameter was 8.9425…, while the answer given in the Sugio Shrine tablet is 8.9425477….

Shortly thereafter, on the 1st of July, Yamaguchi arrived at Akita City [9, p.35] and he visited Koshiō Shrine where he recorded the 16 problems of the sangaku offered by the students of Saitō. On the 22nd of the same month, he went to Tsuruoka City (鶴岡) (where Ōyama mathematics school was located) [9, p.38]. He continued his visits to neighboring cities before arriving in Shinjō City (新庄) on the 27th of July [9, p.39].

5. Yamaguchi Kanzan was the unnamed master

When Endō [1] argued that Saitō Naonaka was the master mentioned in the preamble of the Sugio sangaku, he did not address the fact that the students met this master *recently*, as explicitly stated in the sangaku. Yamaguchi, prior to visiting Tsuruoka, where he probably met some of the students of the Ōyama school on the 22nd of July, had just recorded the problem from Iizuka and Aizawa and the 16 problems from Koshiō Shrine. Nine days later, the Sugio sangaku was hung on the 1st of the August. On a closer inspection of the Sugio sangaku, one can easily see that the handwriting of problem 1 of that sangaku was different from that of the rest of the tablet. This suggests that the sangaku was made in haste, which can be explained by the assumption that a *very recent* visit of this *master* had taken place.

Another unusual characteristic about the Sugio sangaku is that each of the problems was solved with a specified technique as indicated at the beginning of each problem. The three techniques are *yōjutsu* (容術) (see e.g. [8, Ch. 8]), *chikusakujutsu* (逐索術) (see e.g. [8, Ch. 9]), and *tetsujutsu* (綴術), respectively. These are well known techniques in wasan during the Edo period. In particular, *tetsujutsu*[2] is considered one of the highest achievements among the many mathematical methods developed by the masters in the Seki school (see e.g [6]). This method is advanced compared to the elementary method (such as using square roots) that can be employed to solve problem 3. Since the solution for problem 3 (with different numerical values) was also copied in Yamaguchi's diary, it seems that the proposer was aware of the easier method of solving the problem, i.e., using square root, but decided to use an alternate method. It appears that the proposers wished to demonstrate their mathematical prowess. In particular, it is far more complicated

[2]Takebe Katahiro 建部賢弘 (1664 –1739) further developed this technique that allowed him to find infinite series expansion.

to use *tetsujutsu* to solve problem 3 than to use elementary methods using squaring root (explicitly stated not to use). It is conceivable that the approach to solving these 3 problems using the specified techniques might have been suggested by the unnamed master, who would have to be well-trained in these specified techniques. Indeed, Yamaguchi was a 7th generation licensed mathematician of the Seki school.

In conclusion, based upon the timing of Yamaguchi's travel between May and August of 1818 and other evidence, Yamaguchi Kanzan was likely the unnamed master mentioned in the preamble of the Sugio sangaku. Yamaguchi probably showed the 3 problems (of the Sugio sangaku) among other mathematical problems, with answers, to the students of the Ōyama school. He then suggested to the students to try to solve these problems using those specified techniques.

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