6-1-2016

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Recommended Citation
Shea, P. 2016. "Short-sighted managers and learnable sunspot equilibria." Journal of Macroeconomics. 48: 117-126. https://doi.org/10.1016/j.jmacro.2016.02.001

Short-Sighted Managers and Learnable Sunspot Equilibria

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February 2, 2016

Abstract

This paper assumes that firm managers make choices over a finite horizon while households plan over an infinite horizon. Following Shea (2013), I assume that labor exhibits firm-specific learning by doing so that newly employed labor is less productive than experienced labor. In the model, optimization requires that firm managers make conjectures about how their choices affect the labor demand choices of their successors. The model yields two steady states; one where the firm manager behaves as if she cares only about the present period and another where she is forward looking. The former (myopic) steady state usually exhibits higher output than the non myopic steady state. The non-myopic steady state also exhibits two regions of indeterminacy where extraneous, self-fulfilling expectational errors add volatility. One of these regions of indeterminacy is usually stable under adaptive learning while the other never is stable under learning.

Keywords: multiple equlibria, finite horizons, adaptive learning, sunspots.

JEL Classification: D83, D84, E13, E32.

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1 Introduction

Most modern macroeconomic models assume that agents maximize over an infinite horizon. At first glance, finite horizon models, such as the overlapping generations framework, appear to be more plausible. The infinite horizon approach, however, enjoys a strong defense based on the well known result that overlapping generation models with agents who are altruistic toward their children behave identically to infinite horizon models.[1] This result, however, applies far better to households than firms whose managers are unlikely to care about the well-being of their successors. This paper formalizes this distinction by modeling households as maximizing over an infinite horizon, but firm managers as maximizing only over the period where they are being compensated.

The paper's key result is that the model exhibits two distinct types of multiple equilibria. First, there are typically two steady states including a steady state where firm managers behave myopically that exhibits (sometimes dramatically) higher consumption and output than the other, non-myopic steady state. Second, the steady state that usually has lower output frequently exhibits indeterminacy whereby extraneous expectational errors affect the model's dynamics. These sunspots usually add considerable volatility to employment and the wage, and sometimes to output, consumption, and investment as well. Furthermore, this steady state exhibits two distinct regions of indeterminacy. One is usually stable under adaptive learning while the other is always unstable under adaptive learning.

This paper builds on the modeling approach of Shea (2013). That paper makes two modifications to an otherwise ordinary Real Business Cycle (RBC) Model. First, it assumes that labor exhibits firm-specific learning by doing where newly employed labor is less productive than experienced labor. This assumption makes firms' labor demand problem intertemporal and often yields an indeterminate wage rate.[2] By further assuming that firms and households discount at different rates, this indeterminacy also has important effects on quantities in the model, most notably adding considerable volatility to the labor market. That paper simply assumes heterogeneous discount factors and does not explicitly consider different planning horizons as a source of that heterogeneity.[3]

This paper alters Shea (2013) by modeling firm managers as living for only two periods. They work in the first period and are retired in the second. It then analyzes the effects on

[1]See, for example, Barro (1974).

[2]In the New Keynesian setting, firms also face an intertemporal problem which is crucial to generating that literature's main results. See, for example, Woodford (2003).

[3]Assuming heterogeneous discount factors is common in the literature on credit constraints. See, for example, Iacoviello (2005).

the aggregate economy of alternate incentive compatible contracts that potentially compensate firm managers with a share of firm profits over both periods of their lives. As a result, the firm manager no longer faces a standard recursive problem. Instead, optimization requires that they make conjectures about how their labor demand decisions will affect the future stock of experienced (more productive) labor and the decisions of their successors. I borrow the concept of a *consistent conjecture* from an older literature that examines duopoly in Industrial Organization models.[4] I define a consistent conjecture as a Markov perfect Nash Equilibrium where a firm manager expects that her successor will respond to a change in experienced labor just as she herself would.

A key result is that two distinct consistent conjectures exist. Under one, the firm manager behaves as if she maximizes profits in only the first period, even though she is generally compensated with a share of firm profits in her second period of life. I refer to this case as the *myopic steady state*. Under the second, the manager does act as if she maximizes profits over two periods. I refer to this case as the *non-myopic steady state.*

These two steady states exhibit three important differences. First, they yield different levels of output, consumption, employment, etc. This distinction is most dramatic when newly employed labor is relatively unproductive. Here, the myopic steady state exhibit much higher levels of economic activity and lower levels of household utility. Second, while the myopic steady state is always determinate, the non-myopic steady state yields two separate regions of indeterminacy: one where newly employed labor is relatively productive but where second period (of the firm manager's life) compensation is low, and another when newly employed labor is relatively unproductive and where second period compensation is high. Within each of these regions of indeterminacy, extraneous expectational errors destabilize the labor market. In the former region, they also add considerable volatility to output, consumption, and investment.

The model's two steady states differ in a third important aspect. If the extreme informational assumptions of rational expectations are relaxed, and agents are instead assumed to form expectations through adaptive learning, then the rational expectations equilibria are not always learnable in the non-myopic steady state. Under adaptive learning, agents are assumed to estimate the model using least squares. They then use their coefficients to form expectations, and they update these coefficients as new data become available. A solution is learnable if these regression coefficients converge toward their rational expectations values. While the model is learnable whenever it is determinate (in either steady state), it is never learnable in one of the regions of indeterminacy (where newly employed labor is productive) and part of the other

[4]See Bresnahan (1981), Perry (1982), and Dixon and Somma (2003) for microeconomic applications of consistent conjectures.

(where newly employed labor is unproductive).[5] The model is thus unusual in that it yields a large region of indeterminacy where sunspot solutions are learnable.

This model thus yields multiple types of multiple equilibria. The significance of multiple stable steady states is straightforward as these steady states may exhibit important differences over the level of key variables. The most prominent example in macroeconomics is Evans, Honkapohja, and Romer (1998).[6] In that paper, complimentary capital goods and the presence of distinct capital and consumption sectors result in separate steady states that differ in the growth rate. The learning process allows the model to endogenously transition between the neighborhoods of each steady state. In the present paper, the difference between the steady states is over the level, not the growth rate, of output.

1.1 Related Literature on Indeterminacy

It is well known that macroeconomic models may exhibit indeterminacy of equilibrium where a continuum of stable equilibrium paths exist in the neighborhood of a steady state. Indeterminacy has generated considerable interest because random expectational shocks may be self-fulfilling, providing the model with an additional and endogenous source of volatility. These expectational shocks, also known as sunspots, may be viewed as a modern presentation of Keynes's "animal spirits" which he believed importantly contributed to macroeconomic volatility.

Many papers seek to identify plausible assumptions that yield indeterminacy, while also yielding reasonable empirical fit. By far, the most common approach is to assume some type of production externality, or other distortion from complete markets, that causes the aggregate production function to exhibit increasing returns to scale. Early examples include Farmer and Guo (1994), and Benhabib and Farmer (1994), where the degree of increasing returns to scale must be so large that the aggregate labor demand schedule is not only upward sloping, but steeper than labor supply. Additional work finds assumptions that allow for this type of indeterminacy with plausible levels increasing returns to scale. Benhabib and Farmer (1996) introduce a model with distinct sectors producing capital and consumption goods where only small increasing returns to scale are needed. Barinci and Chéron (2001) introduce heterogeneous households who are finance constrained and obtain indeterminacy with a downward sloping labor demand curve. More recently, Meng and Yip (2008) find that the large increasing returns

[5] Throughout the paper, learnability is evaluated using the related concept of E-Stability. Evans and Honkapohja (2001) show that, under general conditions, which apply here, a model is learnable if and only if it is E-Stable.

[6] Macroeconomic models often exhibit a steady state with zero economic activity. These, however, tend to be unstable and are not of great interest.

to scale of Benhabib and Farmer (1994) are not needed and that indeterminacy may occur with a downward sloping labor demand schedule when the intertemporal elasticity of consumption is high and there is a negative capital externality.

The general approach of these papers has been criticized on grounds that go beyond the core assumptions of each specific model. First, the presence of indeterminacy tends to be quite sensitive to parameters in households' utility functions, including the Frisch elasticity of labor supply and the intertemporal elasticity of consumption. In particular, indeterminacy often requires highly inelastic labor supply. Second, indeterminate solutions exist under rational expectations but are almost never stable under adaptive learning. If agents thus form their expectations using common econometric methods, including least squares, then their econometric algorithms will not converge to the rational expectations solution. For those who find adaptive learning to be a desirable approach to modeling expectations formation, these indeterminate solutions are thus not economically plausible.

The present paper generates indeterminacy using a mechanism similar to Shea (2013) that is distinct from the production externality approach. Here, indeterminacy arises because firm managers act in period t on their expectations of firm hiring in period $t+1$ in a manner that makes their expectations self-fulfilling. This mechanism is driven by labor demand and is largely unaffected by labor supply. As a result, the results of this paper are substantively unaffected by altering either the Frisch elasticity of labor supply or the intertemporal elasticity of consumption. Furthermore, because the model yields plausible oscillatory dynamics like Shea (2013), it also generates a robust region where indeterminate solutions are stable under learning. This result is unusual. The only other examples of robust and learnable indeterminacy are McGough, Meng, and Xue (2013), and and Evans and McGough (2005b). The former paper follows Meng and Yip (2008) by assuming assuming a negative production externality and high intertemporal elasticity of consumption in a RBC setup. The latter uses a New Keynesian setup where the monetary authority aggressively responds to both expected inflation and the expected output gap.

There are other mechanisms that yield indeterminacy that are distinct both from that of the present paper and the production externality approach. Indeterminacy often occurs in the voluminous New Keynesian literature, typically when the monetary authority is not sufficiently aggressive in raising rates in response to inflation.[7] The hypothesis that indeterminacy helps explain the excessive volatility of the 1970s enjoys considerable empirical support.[8] Schmitt-Grohé and Uribe (1997) show that indeterminacy can occur when a distortionary tax automatically

[7]See, for example, Woodford (2003).

[8]See Clarida, Gali, and Gertler (2000), Lubik and Shorfheide (2004), and Shea (2008).

adjusts to exogenous changes in government spending in order to balance the government's budget. With the exception of Evans and McGough (2005b), who assume an active as opposed to passive monetary policy, these indeterminate solutions are rarely stable under learning.

The paper is organized as follows. Section 2 develops the model. Section 3 then solves for the multiple steady states and examines their levels, volatility, determinacy, and learnability. Section 4 considers the nature of the efficient employment contract and shows that it exists in only limited circumstances. Furthermore, when it is possible to write a contract that yields the complete markets equilibrium, it may require placing excess weight on future profits that the firm manager obtains in retirement. Section 5 shows that the paper's main results are robust to alternate calibrations. Section 6 concludes.

2 Model

This paper builds on the model of Shea (2013). That paper modifies an ordinary Real Business Cycle Model in two major ways. First, it assumes firm-specific learning by doing whereby newly employed labor becomes more productive if it remains with the same firm for more than one period. This modification causes both labor supply and labor demand to become intertemporal problems and is often sufficient for indeterminacy of the wage rate. Second, it allows firm managers and households to discount at different rates, an assumption that often allows indeterminacy to have effects beyond the wage rate. This paper builds on the latter assumption by formally modeling firm managers as having a finite horizon. As a result, the model now often exhibits multiple steady states as well as indeterminacy.

The representative household consists of a young generation that supplies labor and an old generation that only consumes. After each period, the old member dies, the young member grows old, and a new young member is born. I assume that agents are perfectly altruistic toward their offspring and therefore the household behaves as if it has an infinite horizon when making its ordinary labor supply and consumption choices.

The representative household chooses consumption (C_t) and newly employed labor (N_t). It also supplies managerial effort, denoted Z_t. For the moment, I take this as given.

$$Max_{N_t,C_t,L_{t+1}} \quad E_t \sum_{i=0}^{\infty} \beta^i \left[\frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \gamma \frac{(N_{t+i}+L_{t+i}+Z_{t+i})^{1+\chi}}{1+\chi} \right] \tag{1}$$

s.t.

$$K_t = (1-\delta)K_{t-1} + \kappa w_{t-1}N_{t-1} + w_{t-1}L_{t-1} + r_{t-1}K_{t-1} + \Pi_{t-1} - C_{t-1} + X_{t-1} \tag{2}$$

$$L_{t+1} = (1 - v)(L_t + N_t) \tag{3}$$

The model includes both newly employed (N_t), and experienced labor (L_t). Equation (3) describes the evolution of experienced labor.[9] By assumption, an exogenous fraction, v, of labor separates from its firm each period. The remaining portion may remain with its firm as experienced, more productive labor.

Equation (2) is the budget constraint where w_t is the wage paid to experienced labor and κ is the relative wage paid to newly employed labor. The representative household obtains the same disutility from supplying all types of labor. It obtains income equal to X_t, taken as given for now, in exchange for supplying managerial effort. It receives firm profits (new of managerial compensation) equal to Π_t that it also takes as given.

Optimization yields an Euler Equation and labor supply rule:

$$C_t^{-\sigma} = \beta E_t[(1 - \delta + r_{t+1})C_{t+1}^{-\sigma}] \tag{4}$$

$$\kappa w_t C_t^{-\sigma} + \beta(1 - v)(1 - \kappa)E_t[w_{t+1}C_{t+1}^{-\sigma}] = \gamma(N_t + L_t + Z_t)^{\chi} \tag{5}$$

The household's Euler Equation in consumption, (4) is standard. Its labor supply choice, (5) is not. Suppose that the household were to increase its current supply of newly employed labor by one unit, while also reducing its expected supply of newly employed labor in period $t+1$ by $(1-v)$ units. Such a change would leave periods $t+2$ and beyond unchanged. Equation (5) equates the benefits (increased wages) of such a change to the costs (increased disutility of labor).

I now consider the representative firm's problem. Firms are owned by households. Because there are a large and equal number of both, all households own infinitesimally small shares of each firm. I make the following assumption.

(A1): Because each household owns only an infinitesimally small share of each firm, a manager must be employed to operate each firm. The representative firm manager is drawn from a household and thus lives for two periods, is only employed in the first period, and is retired

[9] This paper assumes that firm managers are part of the representative household but live for only two periods. Such an assumption suggests that the following may be a more sensible specification for experienced labor accumulation: $L_t = (1 - v)N_t$, so that all experienced labor separates from the firm. This alternate approach reduces the steady state values of output, consumption, etc., but does not affect the paper's major results.

in the second period.[10] Because the firm manager is part of a household, she has the discount rate β.

The representative firm manager has only an infinitesimally small ownership stake in the firm she operates. She thus takes aggregate firm profits, Π_t, as given in her budget constraint. She solves her own optimization problem which includes choosing her effort. I make the following assumption.

(A2): The manager must choose managerial effort $Z_t = 0, e$. If she chooses $Z_t = e$, then she obtains the associated disutility and is able to observe market conditions in order to optimally choose the firm's level of employment. If the manager supplies $Z_t = 0$, then she hires no newly employed labor and she rents no capital. For tractability, I further assume that the firm shuts down but must still pay its experienced labor, firm profits thus equal $-L_t w_t$ in period t and zero afterwards.

The manager's compensation is crucial to her optimization problem. I assume the following:

(A3): The manager's effort cannot be observed. The manager's contract consists of a fixed salary ω. In addition, the firm manager also receives a fraction τ of firm-specific profits in the period in which she manages the firm. In order to make the manager care about future firm profits, the firm may also offer her the share of profits $\tau\theta$ in the period in which she is retired where θ is the relative share of profits that the manager receives in the second period compared to the first.

For sufficiently small e, an assumption that will be made when calibrating the model, incentive compatibility requires that:

$$C_t^{-\sigma}\tau\left(\Pi_t + E_t\left[\frac{\beta\theta\Pi_{t+1}}{1-\delta+r_{t+1}}\right] - L_t w_t\right) = \frac{1}{1+\chi}\left[(L_t+N_t+e)^{1+\chi} - (L_t+N_t)^{1+\chi}\right] \tag{6}$$

Equation (6) requires that the increased utility resulting through more consumption due to higher profits must equal the extra disutility from supplying managerial effort.[11]

For the firm manager to participate (as opposed to declining to manage the firm) it must be true that:

[10]The case where the manager lives for only one period is discussed in Section 3.

[11]Equation (6) is an approximation because it does not account for the concavity of the household's utility function. This is not a concern because if e is small, as is imposed when the model is calibrated, then the change in the marginal utility of consumption is also small.

$$\left[\omega+\tau\left(\Pi_t+E_t\left[\frac{\beta\theta\Pi_{t+1}}{1-\delta+r_{t+1}}\right]\right)\right]c_t^{-\sigma}=\frac{1}{1+\chi}\left[(L_t+N_t+e)^{1+\chi}-(L_t+N_t)^{1+\chi}\right] \quad (7)$$

Because the contract includes three parameters (ω, τ, and θ), there exists a continuum of contracts that satisfy (7) and (6). The focus of this paper is on how the choice of θ affects the aggregate economy where the other two parameters are then allowed to adjust to ensure that (7) and (6) hold. As θ increases, the contract induces the firm manager to become more forward looking. The case where $\theta = 0$ is equivalent to the model under the assumption that the firm manager maximizes profits only in the current period.

Because firm managers are also altruistic towards their offspring, a firm could potentially write a contract where the manager is compensated over the infinite horizon by providing all of the manager's heirs with a share of firm profits. Such a contract, appropriately designed, might return the complete markets equilibrium where indeterminacy may still exist, but where its effects are limited to making the wage more volatile. I rule out such contracts for the simple reason that they are rarely, if ever, actually observed.

(A4): For the remainder of the paper, I only examine contracts where the manager receives a share of firm profits while she is still alive.

In general, firms cannot therefore induce their managers to behave as if they have an infinite horizon. Instead, firm managers will maximize profits only over a two period horizon, placing weight on the second period equal to $\hat{\beta} = \beta\theta$.

Because of this approach, the model no longer has a standard recursive structure. When optimizing, the firm manager must now make an assumption about how her choice of newly employed labor affects the choice of her successor. Formally, denote $q = E_t[\frac{\partial N_{t+1}}{\partial L_{t+1}}]$ as the current firm manager's conjecture about how his successor will respond to having an additional unit of experienced labor.

As in Shea (2013), I assume that $MPN_t = \phi MPL_t$, where $\phi < 1$ is thus the relative productivity of newly employed labor. The representative manager's problem then becomes:

$$Max_{N_t} \;\; [Y_t-\kappa w_t N_t - w_t L_t]+\hat{\beta}(1-\delta+r_t)^{-1}E_t[Y_{t+1}-\kappa w_{t+1}N_{t+1}-w_{t+1}L_{t+1}] \quad (8)$$

s.t. (3). Optimization then yields:

$$\kappa w_t+\hat{\beta}(1-v)(1+q\kappa)(1-\delta+r_{t+1})^{-1}E_t[w_{t+1}]=$$

$$\phi MPL_t + \hat{\beta}(1-v)(1+q\phi)(1-\delta+r_{t+1})^{-1}E_t[MPL_{t+1}] \tag{9}$$

$$r_t = MPK_t \tag{10}$$

The notable complication is to solve for the firm manager's conjecture about how her labor supply choice affects that of her successor. To solve for q, I borrow the notion of consistent conjectures from the Industrial Organization literature on duopolies. The firm manager chooses N_t by conjecturing that her successor will choose $\frac{\partial N_{t+1}}{\partial L_{t+1}}$ just as she would choose $\frac{\partial N_t}{\partial L_t}$.

This paper's use of consistent conjectures is neither for simplicity nor tractability. In many settings, representative agents take expectations of aggregate variables as given because they are too small to affect them. Here, the manager cares about firm-specific, not aggregate profits, and her choices do impact firm-specific choice variables in the future. A consistent conjecture is a Markov perfect Nash Equilibrium. It is optimal for the manager to choose $q = \frac{\partial N_t}{\partial L_t}$ as long as all of her successors will behave in the same way. To do otherwise, such as taking $E_t[N_{t+1}]$ as given (setting $q = 0$), would be suboptimal.

To solve for the consistent conjectures, I differentiate (9) with respect to L_t and N_t:

$$\phi^2 f_t^{''} dN_t + \phi f_t^{''} dL_t + \hat{\beta}(1-v)^2 E_t[f_{t+1}^{''}](dN_t + dL_t) + \hat{\beta}q(1-v)^2\phi E_t[f_{t+1}^{''}](dN_t + dL_t) = 0 \tag{11}$$

where $f^{''} = \frac{\partial MPL_t}{\partial L_t}$ and $\hat{\beta} = \theta\beta$, the manager's effective discount factor that is the product of households' discount factor and the manager's share of profits in the second period relative to the first period. When $\theta = 1$, the manager's effective discount factor is the same as the true discount factor. Evaluating (11) at the steady state:

$$q = \frac{\partial N}{\partial L} = -\frac{\phi + (1+q\phi)\hat{\beta}(1-v)^2}{\phi^2 + (1+q\phi)\hat{\beta}(1-v)^2} \tag{12}$$

Equation (12) is a quadratic. Solving yields a pair of steady state consistent conjectures:

$$q^m = -\phi^{-1} \tag{13}$$

$$q^n = -1 - \frac{\phi}{\hat{\beta}(1-v)^2} \tag{14}$$

Inserting (13) into (9) yields:

$$\kappa w_t = \phi MPL_t \tag{15}$$

For this steady state, the firm manager thus behaves as if she cares only about the present period. I thus refer to this steady state as the *myopic steady state*. For the *non-myopic steady state*, represented by (14), the firm manager is forward looking.

Closing the model requires solving for κ. I now make the model's final key assumption:

(A5): Following Shea (2013), κ is determined through a bargaining game between experienced labor and its firm. Because experience is firm-specific, a spot market for experienced labor does not exist. Defining λ as experienced labor's bargaining power, the Nash bargaining solution is thus:

$$\kappa = \frac{\phi}{\lambda + \phi(1-\lambda)} \tag{16}$$

To show that, unlike much of the related literature, indeterminacy requires no increasing returns to scale I assume a production function that exhibits constant returns to scale in labor and capital.

$$Y_t = A_t K_t^{\alpha}(\phi N_t + L_t)^{1-\alpha} \tag{17}$$

$$A_t = A_{t-1}^{\rho}\epsilon_t \tag{18}$$

For either steady state, the model can now easily be linearized and represented as:

$$\begin{bmatrix} \tilde{C}_t \\ \tilde{w}_t \\ \tilde{N}_t \\ \tilde{L}_t \\ \tilde{A}_t \\ \tilde{K}_t \end{bmatrix} = M \begin{bmatrix} \tilde{C}_{t+1} \\ \tilde{w}_{t+1} \\ \tilde{N}_{t+1} \\ \tilde{L}_{t+1} \\ \tilde{A}_{t+1} \\ \tilde{K}_{t+1} \end{bmatrix} + G \begin{bmatrix} \tilde{\epsilon}_{t+1} \\ \tilde{v}_{t+1}^{C} \\ \tilde{v}_{t+1}^{w} \\ \tilde{v}_{t+1}^{N} \end{bmatrix} \tag{19}$$

where $\tilde{v}_{t+1}^{i}$ represents the endogenous expectational error associated with the i^{th} control variable. The model has three such control variables: w_t, N_t, and C_t.

3 Results

The presence of consistent conjectures results in an unusual result. The model now displays two distinct types of multiple equilibria. First, the different conjectures correspond to different steady state levels of output, consumption, etc. Second, the non-myopic steady state frequently exhibits indeterminacy where a continuum of equilibria paths exist around the steady state. These paths may depend on extraneous expectational errors ("sunspots") that are self-fulfilling. Indeterminacy, but not multiple steady states, are a feature of Shea (2013). The myopic steady state, where the firm manager's expectations do not matter, is always determinate.

I begin by comparing the steady state for each type of consistent conjecture. For most parameters, I employ the calibration of Shea (2013), setting $\alpha = 0.36$, $\beta = 0.99$, $\gamma = 1$, $\delta = 0.025$, $\rho = 0.95$, and $\sigma = 1$. Each of these values is common in the literature. I set $\chi = 0$, implying a Frisch intertemporal elasticity of labor supply equal to one and $\upsilon = 0.12$, consistent with data from the Bureau of Labor Statistics on the separation rate of labor. I set $\lambda = \frac{9}{10}$, giving most bargaining power to experienced labor instead of the firms. Finally, I assume that e is sufficiently small so that the manager's participation constraint is always satisfied. Combined, with the calibration that $\chi = 0$ which yields a constant marginal disutility of labor, the firm manager's labor supply choices now have no effect on the model. Section (5) discusses alternate values of χ and σ and shows that they do not affect the paper's conclusions.

I now evaluate the model for alternate values of ϕ and $\hat{\beta}$. The most striking result is that, for low values of ϕ, the myopic steady state exhibits significantly more economic activity than the non-myopic steady state.

Table 1: Steady State Properties, Varying ϕ

ϕ	C^m	C^n	Y^m	Y^n	N^m	N^n	U^m	U^n
0.05	7.57	2.01	8.65	2.30	0.398	0.106	-1.29	-0.185
0.15	2.52	2.04	2.86	2.34	0.131	0.106	-0.168	-0.171
0.25	2.25	2.08	2.57	2.36	0.115	0.107	-0.151	-0.157
0.35	2.19	2.11	2.50	2.41	0.111	0.107	-0.141	-0.144
0.45	2.17	2.15	2.48	2.45	0.109	0.107	-0.129	-0.131
0.55	2.18	2.18	2.49	2.50	0.108	0.108	-0.117	-0.117
0.65	2.20	2.22	2.51	2.54	0.107	0.108	-0.105	-0.104
0.75	2.23	2.27	2.54	2.59	0.107	0.109	-0.092	-0.091
0.85	2.25	2.33	2.57	2.66	0.107	0.111	-0.081	-0.078
0.95	2.28	2.18	2.61	2.49	0.107	0.103	-0.069	-0.074

For low values of ϕ, the myopic steady state includes much higher levels of output, consumption, and employment. Because $\kappa > \phi$, newly employed labor is ordinarily paid more than its marginal product while experienced labor is paid less. Because, the myopic manager only cares about the present, however, she insists that newly employed labor be paid its marginal product. This requires a higher marginal product of labor than in the non-myopic steady state. Surprisingly, this does not entail a lower level of employment. Instead, higher employment and output passes on to a higher capital stock which then increases labor's marginal product. The resulting high level of employment results in a considerable welfare loss, due to excess labor supply, compared to the non-myopic steady state. This result does not hold for higher values of ϕ. As ϕ increases, steady state output in the non-myopic steady state starts to exceed that of the myopic steady state. For ϕ close to one, however, output in the myopic steady state is once against higher.

I now consider the behavior the model in the neighborhood of each steady state. I begin by considering two discrete properties. First, whether or not the model is determinate. The analysis of determinacy is done under the rational expectations assumption following the method of Blanchard and Kahn (1980). Determinacy is easy to evaluate using (19). Each eigenvalue of M inside the unit circle provides a saddle condition that pins down one control variable. The model is thus determinate if there are three such conditions. If there are only two, however, then the model's equilibrium is indeterminate.[12]

Second, I consider whether the model's solution is stable under adaptive learning. Under adaptive learning, agents are assumed to form expectations by estimating the system through least squares regression analysis. In this case, I assume that agents estimate the model using the VAR structure of (19). Agents thus know the model's correct functional form, but unlike rational expectations, they must estimate the values of the coefficients. If the model converges toward rational expectations it is said to be stable under learning. I evaluate stability under learning by examining the related concept of E-Stability. Evans and Honkapohja (2001) show that under general conditions, which apply here, a model is learnable if and only if it is E-Stable. They also provide straightforward conditions to evaluate E-Stability.

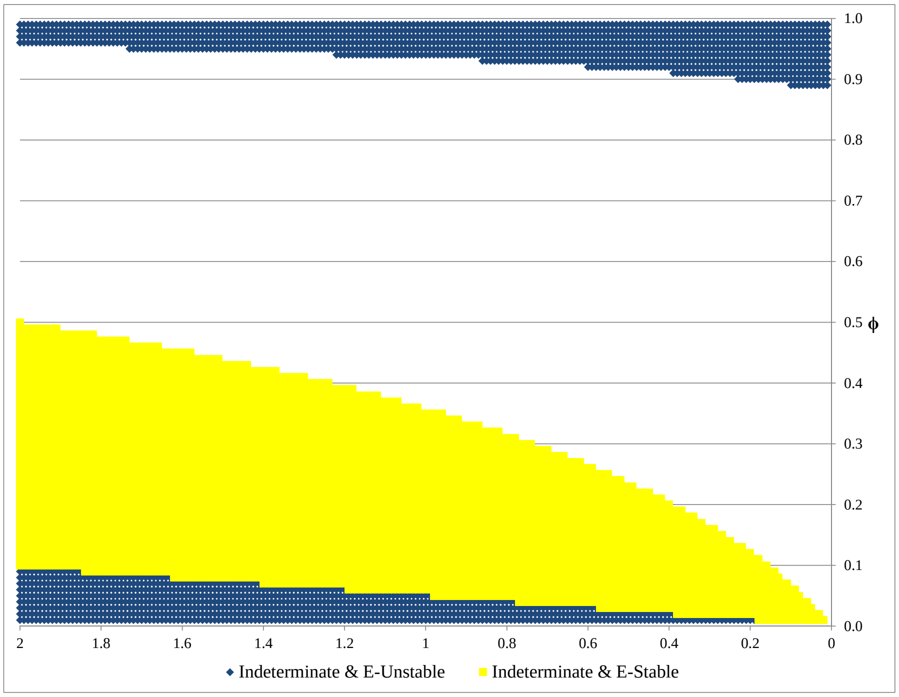
Figure 1 displays the results for the non-myopic steady state:

The model displays two separate regions of indeterminacy. To better understand this result, insert (14) into (9) to obtain the firm's labor demand relationship:

$$w_t = -\left[\frac{(1-\kappa)\hat{\beta}(1-v)}{\kappa} - \frac{\phi}{1-v}\right] E_t[w_{t+1}] + f(MPL_t, E_t[MPL_{t+1}]) \tag{20}$$

here $f(\cdot)$ is a function of the current and expected wage. Determinacy is a property of the

[12]For the calibrated model, there are always either 2 or 3 eigenvalues inside the unit circle. More generally, fewer than three yields indeterminacy. Four or more implies that no stationary solution exists.



$\hat{\beta}$

Figure 1: Regions of Indeterminacy

entire system and cannot be properly evaluated by just looking at (20). Treating the wage terms as exogenous, and evaluating the root from (20) in isolation does, however, provide a helpful approximation where indeterminacy occurs if and only if $|\frac{(1-\kappa)\hat{\beta}(1-v)}{\kappa} - \frac{\phi}{1-v}| > 1$.

One region of indeterminacy covers relatively large values of $\hat{\beta}$ and low values of ϕ (which also implies low values of κ). Note from (20), that for such values, $-(\frac{(1-\kappa)\hat{\phi}(1-v)}{\phi} - \frac{\kappa}{1-v})$ is less than -1. This root results in indeterminacy. Furthermore, as discussed in Evans and McGough (2005a and 2005b), the presence of an additional negative root makes E-Stability more likely. Critically, the conjecture is relatively close to zero, suggesting that when the manager increases N_t her successor decreases N_{t+1} by a fairly small amount. As a result, additional hiring increases the expected wage bill in periods t and $t+1$, resulting in oscillatory dynamics in (20). Sunspots then serve to increase wages in $t+1$ while also decreasing them in t, or vice-versa. If households and managers discount in the same way, all agents are indifferent to these fluctuations, the wage may be eliminated from the system, and the effect of indeterminacy is limited to a more volatile wage. If managers place different weight on period $t+1$ than households, however, then the wage may not be eliminated and indeterminacy has real effects. This case is similar to Shea (2013).

The second area of indeterminacy occurs for relatively high values of ϕ and low values of

$\hat{\beta}$. From, (20), this root is greater than one. Here, the conjecture is far from zero indicating that when when the manager increases N_t, her successor decreases N_{t+1} by so much that the expected wage bill in period $t+1$ actually declines. Now, a sunspot causes w_t and $E_t[w_{t+1}]$ to move in the same direction. As is usually the case when indeterminacy results from a positive root, these indeterminate solutions are not stable under learning.

I now compare the model's behavior in neighborhood of each steady state under rational expectations. As discussed earlier, the myopic steady state is always determinate and is thus unaffected by sunspots. The non-myopic steady state, however, exhibits distinct regions of indeterminacy for sufficiently low, and sufficiently high values of ϕ.

Table 2: Standard Deviations as a % of Steady State Values, Varying ϕ

ϕ	C^m	C^n	Y^m	Y^n	N^m	N^n	I^m	I^n	Det^n
0.05	0.82	1.36	1.18	1.70	1.58	1.33	3.51	4.01	Yes
0.15	1.36	1.39	1.73	1.73	1.86	1.53	4.28	4.06	Yes
0.25	1.38	1.40	1.73	1.74	1.63	2.87	4.15	4.10	Yes
0.35	1.38	1.39	1.73	1.73	1.58	1.47	4.13	4.09	No
0.45	1.37	1.38	1.73	1.73	1.55	1.50	4.12	4.11	No
0.55	1.39	1.39	1.73	1.74	1.52	1.53	4.11	4.12	No
0.65	1.39	1.38	1.74	1.73	1.48	1.56	4.11	4.14	No
0.75	1.40	1.39	1.75	1.75	1.44	1.61	4.12	4.19	No
0.85	1.39	1.36	1.73	1.72	1.39	1.72	4.10	4.24	No
0.95	1.39	2.01	1.73	2.42	1.35	5.99	4.10	5.46	Yes

Three results stand out. First, for $\phi = 0.05$, the myopic steady state exhibits much greater stability along with its inefficiently high levels of output and consumption. Second, the effects of the low ϕ type of indeterminacy are similar as in Shea (2013). Indeterminacy destabilizes the labor market, but has only small affects on the volatilities of output and consumption. Third, indeterminacy has much more significant effects for the high ϕ region of indeterminacy. Here, sunspots significantly destabilize output, consumption, and investment. This last result, however, requires a caveat. Table 2 is calculated under rational expectations. As shown in Figure 1, however, the model's solution is not stable under learning. There is thus no plausible means for the model to converge to its rational expectations solution. The dynamics of the model are thus very likely far more volatile in this case than suggested by Table 2.

I now consider the effects of altering the manager's contract by changing $\hat{\beta}$. Throughout this exercise, I hold newly employed labor's productivity constant by setting $\phi = 0.3$.

For $\phi 0.3$, the myopic steady state exhibits higher output, consumption, and utility than the non-myopic steady state. As seen in Table 1, however, this result does not hold throughout the parameter space. The case where $\hat{\beta} = 0.99$ is of special interest. This is the contract

Table 3: Steady State Properties, Varying $\hat{\beta}$

$\hat{\beta}$	C^m	C^n	Y^m	Y^n	N^m	N^n	U^m	U^n
0.19	2.06	2.05	2.29	2.29	0.105	0.105	-0.153	-0.153
0.39	2.10	2.07	2.34	2.31	0.107	0.106	-0.150	-0.152
0.59	2.15	2.09	2.40	2.32	0.110	0.106	-0.148	-0.151
0.79	2.20	2.09	2.45	2.35	0.112	0.107	-0.146	-0.151
0.99	2.26	2.10	2.52	2.34	0.115	0.107	-0.145	-0.151
1.19	2.32	2.10	2.58	2.34	0.118	0.107	-0.144	-0.150
1.39	2.39	2.11	2.66	2.35	0.122	0.107	-0.144	-0.150
1.59	2.45	2.11	2.74	2.35	0.126	0.107	-0.145	-0.150
1.79	2.53	2.11	2.82	2.35	0.129	0.108	-0.147	-0.150
1.99	2.62	2.11	2.92	2.35	0.134	0.108	-0.150	-0.150

that induces the firm manager to discount future profits at the same rate as the representative household discounts future utility. This contract does nothing to eliminate the possibility of multiple steady states. It also does not perform best under either steady state. Under the myopic steady state, a slightly higher $\hat{\beta}$ does better. Under the non-myopic steady state, utility slowing increases along with $\hat{\beta}$ up to $\hat{\beta} = 1.99$.

I now consider the dynamics around each steady state for different values of $\hat{\beta}$:

Table 4: Standard Deviations as a % of Steady State Values, Varying $\hat{\beta}$

$\hat{\beta}$	C^m	C^n	Y^m	Y^n	N^m	N^n	I^m	I^n	Det^n
0.19	1.39	1.38	1.73	1.72	1.51	1.51	3.78	4.08	Yes
0.39	1.39	1.39	1.73	1.73	1.54	1.48	3.87	4.08	Yes
0.59	1.38	1.39	1.73	1.73	1.57	1.47	3.95	4.08	Yes
0.79	1.39	1.39	1.74	1.74	1.60	9.13	4.04	4.16	No
0.99	1.39	1.39	1.73	1.73	1.63	2.28	4.13	4.08	No
1.19	1.38	1.38	1.74	1.72	1.67	1.44	4.25	4.06	No
1.39	1.38	1.40	1.73	1.74	1.71	1.76	4.38	4.09	No
1.59	1.37	1.39	1.74	1.73	1.77	2.12	4.52	4.08	No
1.79	1.37	1.38	1.73	1.72	1.82	2.41	4.65	4.09	No
1.99	1.36	1.39	1.73	1.73	1.88	2.63	4.82	4.09	No

The most striking result from Table 4 is that, as the model enters the region of indeterminacy in the non-myopic steady state, the volatility of the labor market increases dramatically. This is similar to Shea (2013).

4 The First Best Contract

I now examine whether the firm can write a contract with managers that yields the complete markets equilibrium. Suppose that households themselves could choose firms' labor demand. The problem is now recursive and optimality yields:

$$\kappa w_t + (1-v)(1-\kappa)(1-\delta+r_{t+1})^{-1}E_t[w_{t+1}] =$$

$$\phi MPL_t + (1-v)(1-\phi)(1-\delta+r_{t+1})^{-1}E_t[MPL_{t+1}] \tag{21}$$

For exposition, note that at the steady state, $(1-\delta+r_{t+1})^{-1} = \beta$. Making this substitution:

$$\kappa w_t + \beta(1-v)(1-\kappa)E_t[w_{t+1}] =$$

$$\phi MPL_t + \beta(1-v)(1-\phi)(1-\delta+r_{t+1})^{-1}E_t[MPL_{t+1}] \tag{22}$$

To see how this problem is recursive, suppose that the firm increases N_t by one unit and then expects to decrease N_{t+1} by $1-v$ units. Using (3), the firm's stock of experienced labor is unaffected in periods $t+2$ and beyond. The left hand side of (22) captures the costs of such a change: it includes the costs of employing more newly employed labor in t and more experienced labor in $t+1$. The right hand side includes increased output in both periods. Because such a re-arrangement of labor demand is feasible, optimality requires that it cannot change profits.

In the model of Section 2, however, this analysis does not apply. Although this re-arrangement of labor demand is still feasible, the current manager has no reason to believe that the next manager will choose N_{t+1} in such a way to make it actually happen. Thus the problem is not a standard recursive one and the current manager must therefore rely on her conjecture about how her successor will choose N_{t+1}.

The optimal contract is one that sets $\hat{\beta}$ so that (9) and (22) are identical. This requires both:

$$\hat{\beta} = \frac{\beta(1-\kappa)}{1-q\kappa} \tag{23}$$

$$\hat{\beta} = \frac{\beta(1-\phi)}{1-q\phi} \tag{24}$$

The existence of such a contract requires two conditions. First, $\kappa = \phi$ so that labor is always paid its marginal product. Using (16), this entails giving experienced labor all of the bargaining power so that it is able to capture all of its additional productivity over newly employed labor. Second, it requires that the economy chooses the non-myopic steady state, otherwise $\hat{\beta}$ approaches infinity.

For the case where $\kappa = \phi$, and the calibration setting $\phi = 0.3$ and $\upsilon = 0.12$, solving for (23) yields $\hat{\beta} = 1.41$. Notably, the first best contract thus requires that firm managers receive a larger share of firm profits while they are retired than when they actually operate the firm. Also, this calibration yields indeterminacy. Here, indeterminacy affects only the wage rates and has no effects on any real variables. The result is thus consistent with the well established result of Cass and Shell (1983) that shows that indeterminacy may not have real effects in a model with complete markets.

5 Robustness to Alternate Calibrations

There is no consensus about the proper calibration of σ, the intertemporal elasticity of substitution, or χ, the inverse Frisch elasticity of labor supply.[13] This uncertainty is especially relevant for a paper on indeterminacy because other mechanisms that result in indeterminacy tend to be sensitive to the calibrations of these parameters. Relatively inelastic labor supply often eliminates indeterminacy in models similar to Farmer and Guo (1994), and the results of Meng and Yip (2008), and McGough, Meng, and Xue (2013) require a relatively high, though not implausible, value of σ. This concern does not apply to this paper. Consider the first best contract. Here, the wage rate may be eliminated from the system so that the marginal product of labor is exogenous in (22). The indeterminacy of the wage now depends entirely on β, κ, and υ, and is independent of χ and σ. Deviating from the first best contract, or focusing on cases where it is infeasible, does not add any channel by which these latter two parameters are important to indeterminacy.

To confirm this intuition, I examine alternate calibrations where $\chi = 5$ and $\chi = 10$. The two distinct steady states continue to exist with one exhibiting both myopia and determinacy. For the non-myopic steady state, for these two alternate calibrations, the effect on the region of E-unstable indeterminacy is negligible. The size of the region of E-stable indeterminacy does decrease but only very slightly by 0.30% and 0.39% respectively, relative to its size for the baseline calibration.

[13] See Chetty, Guren, Manoli, and Weber (2012) for details on the debate over χ. Guvenen (2006) provides a similar discussion for the debate over σ.

The results are similar when I consider alternate cases where $\sigma = 4$ and $\sigma = \frac{1}{4}$. The only effect is again on the size of the region of E-stable indeterminacy which changes very slightly by 2.1% and -0.07% respectively. Collectively, these results show that neither χ nor σ is crucial to this paper's major conclusions.

6 Conclusion

A fundamental difference exists between firm managers, who are unlikely to be altruistic toward their successor, and households, who are much more likely to be altruistic toward their offspring. This paper has formalized this distinction by imposing a finite horizon on the former and an infinite horizon on the latter. It then examines the effects of alternate contract.

The most striking feature is that the resulting model is prone to exhibiting both multiple steady states and indeterminacy around one of these steady states. One might expect it to be possible to always design a contract that incentivizes the firm manager to discount in the same manner as the household. But this is not the case. Even if $\hat{\beta} = 0.99$, so that firms and households have the same discount factor, the myopic steady state continues to exist. Furthermore, the myopic steady state often performs better than the non-myopic steady state.

This paper also contributes to the small set of papers showing that reasonable assumptions can yield sunspot solutions that are stable under adaptive learning. This allows the model to exhibit additional volatility as sunspots may have real effects. Furthermore, when two steady states exist that are stable under learning, agent's initial conditions or specific learning algorithms may determine which one the economy converged towards. Examining this process in detail is worthy of future research.

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