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**Shedding New Light on Weak Emission-line Quasars in the C iv–H β Parameter Space**

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Shedding New Light on Weak Emission-line Quasars in the CIV–Hβ Parameter Space

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Abstract
Weak emission-line quasars (WLQs) are a subset of type 1 quasars that exhibit extremely weak Lyα + N V λ1240 and/or C IV λ1549 emission lines. We investigate the relationship between emission-line properties and accretion rate for a sample of 230 “ordinary” type 1 quasars and 18 WLQs at z < 0.5 and 1.5 < z < 3.5 that have rest-frame ultraviolet and optical spectral measurements. We apply a correction to the Hβ-based black hole mass (MBH) estimates of these quasars using the strength of the optical FeII emission. We confirm previous findings that WLQs’ MBH values are overestimated by up to an order of magnitude using the traditional broad-emission-line region size–luminosity relation. With this MBH correction, we find a significant correlation between Hβ-based Eddington luminosity ratios and a combination of the rest-frame CIV equivalent width and CIV blueshift with respect to the systemic redshift. This correlation holds for both ordinary quasars and WLQs, which suggests that the two-dimensional C IV parameter space can serve as an indicator of accretion rate in all type 1 quasars across a wide range of spectral properties.

Unified Astronomy Thesaurus concepts: Quasars (1319); Active galactic nuclei (16); Supermassive black holes (1663)

Supporting material: machine-readable table

1. Introduction
Weak emission-line quasars (WLQs) are a subset of active galactic nuclei (AGNs) with extremely weak or undetectable rest-frame UV emission lines (e.g., Fan et al. 1999; Anderson et al. 2001; Collinge et al. 2005; Plotkin et al. 2010). The Sloan Digital Sky Survey (SDSS; York et al. 2000) has discovered ≈103 type 1 quasars with Lyα + N V λ1240 rest-frame equivalent width (EW) < 15.4 Å and/or C IV λ1549 EW < 10.0 Å (e.g., Diamond-Stanic et al. 2009; Meusinger & Balafkan 2014). These numbers represent a highly significant concentration of quasars at >3σ deviation from the log-normal EW distribution of the SDSS quasar population, with no corresponding “tail” at the opposite end of the distribution (Diamond-Stanic et al. 2009; Wu et al. 2012). Furthermore, the fraction of WLQs among the broader quasar population increases sharply at higher redshifts (and thus higher luminosities), from ~0.1% at 3 < z < 5 to ~10%–15% at z > 5.7 (Diamond-Stanic et al. 2009; Bañados et al. 2016; Shen et al. 2019).

Multiwavelength observations of sources of this class have shown that they are unlikely to be high-redshift galaxies with apparent quasar-like luminosity due to gravitational-lensing amplification, dust-obscured quasars, or broad absorption line (BAL) quasars (e.g., Shemmer et al. 2006, 2010) but that their UV emission lines are intrinsically weak. Furthermore, WLQs are typically radio quiet and have X-ray and mid-infrared properties inconsistent with those of BL Lac objects.
(Shemmer et al. 2009; Lane et al. 2011; Wu et al. 2012; Massaro et al. 2017).

About half of WLQs have notably lower X-ray luminosities than expected from their monochromatic luminosities at 2500 Å (e.g., Luo et al. 2015; Ni et al. 2018, 2022; Timlin et al. 2020). One explanation for this phenomenon is that, at small radii, the geometrically thick accretion disks of these WLQs are "puffed up" and prevent highly ionizing photons from reaching the broad emission-line region (BELR; e.g., Wu et al. 2011, 2012; Luo et al. 2015; Ni et al. 2018, 2022). The X-ray radiation is partially absorbed by the thick disk, resulting in low apparent X-ray luminosities at high inclinations (i.e., when these objects are viewed edge on). When these objects are viewed at much lower inclinations, their notably steep X-ray spectra indicate accretion at a high Eddington luminosity ratio ($L_{\text{bol}}/L_{\text{Edd}}$; hereafter, Blueshift et al. 2009, McCaffrey & Richards 2021).

Dong et al. 2009 particularly in quasars with strong optical FeII emission. Hereafter R22.

The structure of this paper is as follows. In Section 2, we discuss our sample selection and the relevant equations used to estimate $H\beta$-based $L/L_{\text{Edd}}$ values. In Section 3, we analyze the samples’ spectroscopic properties as well as the sources’ black hole masses and accretion rates. Subsequently, we discuss the correlation between the CIV parameter space and $L/L_{\text{Edd}}$. In Section 4, we summarize our findings. Throughout this paper, we compute luminosity distances using a standard ΛCDM cosmology with $H_0 = 70 \text{ km s}^{-1} \text{Mpc}^{-1}$, $\Omega_M = 0.3$, and $\Omega_\Lambda = 0.7$ (e.g., Spergel et al. 2007).

2. Sample Selection and Data Analysis

2.1. WLQ Sample

We compile a sample of 18 WLQs that have accurate full-width-at-half-maximum intensity of the broad component of the $H\beta$ λ4861 emission line (hereafter, FWHM($H\beta$)), monochromatic luminosity at rest-frame 5100 Å (hereafter, $L_5100$), EW(Fe II λλ4434–4684), and EW($H\beta$) measurements. Nine of these sources were obtained from SL15, seven from the GNIRS-DQ5 sample of Paper I (see Section 2.2), and two from this work (see the Appendix). SL15 compiled a sample of nine WLQs: SDSS J0836+1425, SDSS J1411+1402, SDSS J1417+0733, SDSS J1447–0203 (Plotkin et al. 2010, 2015), SDSS J0945+1099 (Hryniewicz et al. 2010; Plotkin et al. 2015), SDSS J1141+0219, SDSS J1237+6301 (Diamond-Stanic et al. 2009; Shemmer et al., 2010), SDSS J1521+5202 (Just et al. 2007; Wu et al. 2011), and PHL 1811 (Leighly et al. 2007).

Table 1 provides basic properties for the 18 WLQs in our sample. Column (1) provides the source name; Column (2) gives the systemic redshift determined from the peak of, in order of preference, the [O III] λ5007, Mg ii λ2798, and Hβ emission lines; Column (3) gives log $L_5100$ (5100 Å); Column (4) gives FWHM ($H\beta$); Column (5) gives $R_{Fe ii} \approx$ EW(Fe II)/EW(H$\beta$); Column (6) gives traditional $H\beta$-based $M_{BH}$ estimates (following Equations (2) and (4)); Column (7) gives Fe II-corrected $H\beta$-based $M_{BH,corr}$ estimates (following Equations (3) and (4)); Column (8) gives traditional $H\beta$-based $L/L_{\text{Edd}}$ values (from Equation (5)); Column (9) gives Fe II-corrected $H\beta$-based $L/L_{\text{Edd,corr}}$ values (from Equation (5)); and Column (10) gives EW(CIV). Column (11) gives BlueShift(CIV). Columns (12) and (13) provide the references for the rest-frame optical and UV spectral measurements, respectively. All derived properties are discussed in detail in Section 2.4.

In this work, we explore two possible explanations for the findings of SL15. The first of these is that the traditional estimation of $H\beta$-based black hole mass ($M_{BH}$) values, and therefore $L/L_{\text{Edd}}$ values, fails to accurately predict $M_{BH}$, particularly in quasars with strong optical Fe II emission (e.g., Shen 2013; Mathil et al. 2022). Such a case is typical for WLQs, and thus a correction via measurement of the strength of the Fe II emission complex in the optical band is required (Du & Wang 2019; Yu et al. 2020b). The second explanation is that EW(CIV), by itself, is not an ideal indicator of the quasar accretion rate. In addition to EW(CIV), we utilize a recently defined parameter, the “CIV || Distance” (Rivera et al. 2022, hereafter R22), which represents a combination of the EW(CIV) and BlueShift(CIV) (Richards et al. 2011; Rivera et al. 2020; McCaffrey & Richards 2021), and search for a correlation between that parameter and $L/L_{\text{Edd}}$.
Table 1
Basic Properties of the WLQ Sample

<table>
<thead>
<tr>
<th>Quasar</th>
<th>$z_{\text{sys}}$</th>
<th>log $\nu L_\nu$(5100 Å) (erg s$^{-1}$)</th>
<th>FWHM(H$\beta$) (km s$^{-1}$)</th>
<th>$R_{\text{Fe II}}$</th>
<th>log $M_{\text{BH}}$ ($M_\odot$)</th>
<th>log $M_{\text{BH,corr}}$ ($M_\odot$)</th>
<th>$L/L_{\text{Edd}}$</th>
<th>$L/L_{\text{Edd,corr}}$</th>
<th>EW(C IV) (Å)</th>
<th>Bluelshift(C IV) (km s$^{-1}$)</th>
<th>Optical Referencesa</th>
<th>C IV Referencesa</th>
</tr>
</thead>
<tbody>
<tr>
<td>SDSS J010643.23–031536.4</td>
<td>2.248</td>
<td>46.51</td>
<td>6782</td>
<td>0.58</td>
<td>9.99</td>
<td>9.71</td>
<td>0.20</td>
<td>0.39</td>
<td>7.6$^{+0.6}_{-0.6}$</td>
<td>1451$^{+119}_{-98}$</td>
<td>1</td>
<td>2</td>
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<tr>
<td>SDSS J013136.44+130331.0</td>
<td>1.599</td>
<td>46.45</td>
<td>2294</td>
<td>0.78</td>
<td>9.02</td>
<td>8.67</td>
<td>1.63</td>
<td>3.67</td>
<td>2.8$^{+1.4}_{-2.0}$</td>
<td>2320$^{+519}_{-521}$</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>SDSS J013417.81–005036.2</td>
<td>2.270</td>
<td>46.45</td>
<td>5211</td>
<td>0.98</td>
<td>9.73</td>
<td>9.31</td>
<td>0.32</td>
<td>0.84</td>
<td>7.3$^{+0.7}_{-0.6}$</td>
<td>2233$^{+651}_{-414}$</td>
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<td>2</td>
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<td>SDSS J075115.43+505439.1</td>
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<td>46.59</td>
<td>3077</td>
<td>3.05</td>
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<td>8.19</td>
<td>1.05</td>
<td>15.04</td>
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<td>5953$^{+724}_{-342}$</td>
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<td>2</td>
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<td>SDSS J083650.86+142539.0</td>
<td>1.749</td>
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<td>2880</td>
<td>2.48</td>
<td>8.94</td>
<td>8.04</td>
<td>0.62</td>
<td>4.95</td>
<td>4.2$^{+0.3}_{-0.5}$</td>
<td>2266$^{+191}_{-191}$</td>
<td>3</td>
<td>3</td>
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<tr>
<td>SDSS J085337.36+121800.3</td>
<td>2.197</td>
<td>46.56</td>
<td>4502</td>
<td>0.28</td>
<td>9.66</td>
<td>9.48</td>
<td>0.47</td>
<td>0.73</td>
<td>7.7$^{+1.1}_{-1.7}$</td>
<td>1166$^{+632}_{-342}$</td>
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<td>2</td>
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<td>SDSS J085344.17+354104.5</td>
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<td>46.40</td>
<td>4168</td>
<td>0.72</td>
<td>9.51</td>
<td>9.18</td>
<td>0.47</td>
<td>1.00</td>
<td>4.3$^{+0.8}_{-1.2}$</td>
<td>2053$^{+380}_{-1094}$</td>
<td>1</td>
<td>2</td>
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<tr>
<td>SDSS J094533.98+100950.1</td>
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<td>46.17</td>
<td>4278</td>
<td>2.00</td>
<td>9.41</td>
<td>8.66</td>
<td>0.35</td>
<td>2.03</td>
<td>2.9$^{+0.3}_{-0.4}$</td>
<td>5485$^{+380}_{-380}$</td>
<td>3</td>
<td>3</td>
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<td>SDSS J094602.31+274407.0</td>
<td>2.488</td>
<td>46.75</td>
<td>3833</td>
<td>1.65</td>
<td>9.63</td>
<td>8.94</td>
<td>0.79</td>
<td>3.82</td>
<td>5.9$^{+0.4}_{-0.6}$</td>
<td>9062$^{+16}_{-11}$</td>
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<td>2</td>
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<tr>
<td>SDSS J113747.64+391941.5</td>
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<td>45.81</td>
<td>2518</td>
<td>3.31</td>
<td>8.76</td>
<td>7.57</td>
<td>0.72</td>
<td>10.99</td>
<td>8$^{+3}_{-5}$</td>
<td>3089$^{+250}_{-236}$</td>
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<td>SDSS J114153.33+021924.4</td>
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<td>46.55</td>
<td>5900</td>
<td>3.25</td>
<td>9.89</td>
<td>8.67</td>
<td>0.27</td>
<td>4.60</td>
<td>0.4$^{+0.4}_{-0.4}$</td>
<td>$-777^{+261}_{-188}$</td>
<td>5</td>
<td>6.4</td>
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<tr>
<td>SDSS J123743.07+630144.7</td>
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<td>46.35</td>
<td>5200</td>
<td>2.86</td>
<td>9.68</td>
<td>8.61</td>
<td>0.29</td>
<td>3.39</td>
<td>1$^{+2}_{-2}$</td>
<td>$-973^{+1249}_{-513}$</td>
<td>5</td>
<td>4</td>
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<tr>
<td>SDSS J141141.96+140233.9</td>
<td>1.754</td>
<td>45.64</td>
<td>3966</td>
<td>1.41</td>
<td>9.06</td>
<td>8.56</td>
<td>0.24</td>
<td>0.78</td>
<td>3.8$^{+0.8}_{-0.2}$</td>
<td>3142$^{+529}_{-328}$</td>
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<td>2</td>
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<tr>
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<td>45.91</td>
<td>2784</td>
<td>1.65</td>
<td>8.90</td>
<td>8.29</td>
<td>0.65</td>
<td>2.64</td>
<td>2.5$^{+2.1}_{-0.7}$</td>
<td>5321$^{+4178}_{-344}$</td>
<td>1</td>
<td>2</td>
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<tr>
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<td>1923</td>
<td>1.60</td>
<td>8.39</td>
<td>7.83</td>
<td>0.96</td>
<td>3.52</td>
<td>7.7$^{+0.2}_{-0.3}$</td>
<td>1319$^{+339}_{-339}$</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>SDSS J152156.48+520238.5</td>
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<td>47.14</td>
<td>5750</td>
<td>1.64</td>
<td>10.19</td>
<td>9.48</td>
<td>0.52</td>
<td>2.69</td>
<td>9.1$^{+0.6}_{-1.1}$</td>
<td>4900$^{+700}_{-400}$</td>
<td>7</td>
<td>7</td>
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<td>SDSS J213742.25–003912.7</td>
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<td>7.89</td>
<td>0.62</td>
<td>4.68</td>
<td>3$^{+2}_{-1}$</td>
<td>4986$^{+467}_{-398}$</td>
<td>4</td>
<td>4</td>
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<td>PHL 1811</td>
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<td>45.56</td>
<td>1943</td>
<td>1.29</td>
<td>8.40</td>
<td>7.94</td>
<td>0.94</td>
<td>2.70</td>
<td>6.6$^{+0.4}_{-0.9}$</td>
<td>1400$^{+250}_{-250}$</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

Notes.

a Sources of rest-frame optical–UV data, Column (12): $z_{\text{sys}}$, log $\nu L_\nu$(5100 Å), FWHM(H$\beta$), $R_{\text{Fe II}}$, Column (13): EW(C IV), and Bluelshift(C IV). (1) Paper I, (2) Paper II, (3) Plotkin et al. (2015), (4) this work, (5) Shemmer et al. (2010), (6) Shen et al. (2011), (7) Wu et al. (2011), (8) Leighly et al. (2007).

b Wu et al. (2011) also reported H$\beta$-based Bluelshift(C IV) = 9400 km s$^{-1}$. Here, we have opted to use a Mg II-based value of Bluelshift(C IV).

c Leighly et al. (2007) reported the $R_{\text{Fe II}}$ value as being in the range 1.22–1.35. We have adopted a mean value of 1.29 for this work.
peak of the emission line; the FWHM is restricted to not exceed 15,000 km s\(^{-1}\). Furthermore, we visually inspect the initial fit to correct for any additional residuals. The EW of the line emission can then be measured, as well as the blueshift, which is calculated from the difference between \(\lambda 1549\) and the rest-frame wavelength of the peak of the emission-line profile (see Equation (1)).

Our WLQs appear to possess stronger relative optical Fe II emission (indicated by the larger \(R_{\text{Fe II}}\) values) compared to ordinary quasars from their respective samples. Since such sources are selected based only on their [C IV] emission-line strength (EW([C IV]) < 10 Å), we are unable to assess any potential biases introduced by the rest-frame optical emission to their selection process.

2.2. Ordinary Quasar Sample Selection

In order to create a comprehensive comparison sample of quasars for our analysis, which requires measurements of both the Hβ and [C IV] emission lines, we select two catalogs of ordinary quasars from the literature. For the high-redshift quasars (1.5 \(\leq z \leq 3.5\)), [C IV] emission properties can be obtained from SDSS, but the Hβ emission line lies outside of the SDSS range, and therefore it has to be measured with NIR spectroscopy. In this redshift range, we utilize the GNIRS-DQS catalog in Paper I. GNIRS-DQS is the largest and most comprehensive inventory of rest-frame optical properties for luminous quasars, notably the Hβ, [O III], and Fe II emission lines. To complement this sample of high-redshift, high-luminosity quasars, we include an archival sample of quasars in the low-redshift regime from the BL04 subsample also utilized in SL15. In this redshift range (\(z < 0.5\)), the Hβ emission properties can be obtained from optical spectra, but the [C IV] emission-line properties are more difficult to obtain and are available primarily from the Hubble Space Telescope (HST) and the International Ultraviolet Explorer (IUE) archives. Below, we briefly discuss the selection process for our ordinary quasar sample.

The GNIRS-DQS sources were selected to lie in three narrow redshift intervals, 1.55 \(\leq z \leq 1.65\), 2.10 \(\leq z \leq 2.40\), and 3.20 \(\leq z \leq 3.50\) to center the Hβ+[O III] spectral complex in the NIR bands covered by GNIRS (i.e., the J, H, and K bands, respectively). In total, the survey comprises 260 sources with high-quality NIR spectra and comprehensive Hβ, [O III], and Fe II spectral measurements (see Matthews et al. 2021 and Paper I for more details). We exclude 64 BAL quasars, 16 radio loud quasars (RLQs), and one quasar, SDSS J114705.24+083900.6, that is both BAL and radio loud. We define RLQs as sources having radio-loudness values of \(R > 100\) (where \(R\) is the ratio between the flux densities at 5 GHz and 4400 Å; Kellermann et al. 1989). RLQs and BAL quasars are excluded to minimize the potential effects of continuum boosting from a jet (e.g., Meusinger & Balafkan 2014) and absorption biases (e.g., see BL04), respectively. Two quasars, SDSS J073132.18+461347.0 and SDSS J141617.38+264906.1, are excluded due to a lack of [C IV] measurements from Paper II. In total, 177 GNIRS-DQS quasars are included in our analysis; of these, seven sources with EW([C IV]) \(< 10\) Å can be formally classified as WLQs (see Section 2.1). We adopt values of FWHM(Hβ), \(\nu L_\beta(5100\) Å), EW(Hβ), and EW(Fe II) values from Paper I. The latter two parameters are used to derive \(R_{\text{Fe II}}\). Paper II reports the EW([C IV]) values and the wavelengths of the [C IV] emission-line peaks for the quasars in Paper I, which are then used to derive the Blueshift([C IV]) values (see Section 2.3).

Sixty quasars at \(z < 0.5\) from BL04 are added to our analysis from the 63 BL04 quasars in SL15. PG 0049+171, PG 1427+480, and PG 1415+451 are excluded due to a lack of published Fe II spectral measurements. The UV data in the BL04 sample comes, roughly equally, from both the HST and the IUE archives (see Baskin & Laor 2005). Throughout this work, we check whether including only HST or IUE data changes the conclusion of the paper, but we find no statistical difference in the results of Section 3. Therefore, we include both subsets in this work. We obtain the FWHM(Hβ), \(\nu L_\beta(5100\) Å), and \(R_{\text{Fe II}}\) values for the BL04 sources from Boroson & Green (1992) and their EW([C IV]) and Blueshift ([C IV]) values from Baskin & Laor (2005). The line measurements are expected to be roughly consistent across the different samples utilized in this work since they all employed similar standard fitting procedures. Table 2 lists the basic properties of the ordinary quasars in our sample with the same formatting as Table 1.

2.3. Systemic Redshifts and Blueshift([C IV])

We derive the Blueshift([C IV]) values of GNIRS-DQS sources from the observed wavelengths of the [C IV] emission-line peaks reported in Paper II and the systemic redshifts reported in Paper I. The Blueshift([C IV]) values are derived following Equation (2) in Dix et al. (2020)

\[
\frac{\Delta v}{\nu_{\text{meas}} - \nu_{\text{sys}}} = \left[ \frac{e}{\nu_{\text{meas}} - \nu_{\text{sys}}} \right] \left( \frac{\nu_{\text{meas}} - \nu_{\text{sys}}}{1 + \nu_{\text{sys}}} \right) - \Delta \nu \]

where \(\nu_{\text{meas}}\) is the redshift measured from the wavelength of the [C IV] emission-line peak and \(\nu_{\text{sys}}\) is the systemic redshift with respect to the [O III], the Mg II, or the Hβ emission lines reported in Paper I. In this work, we report the Blueshift([C IV]) \(\equiv -\Delta \nu\) values.

A nonnegligible fraction (\(\sim 1/3\)) of luminous quasars have extremely weak or undetectable [O III] emission (e.g., Netzer et al. 2004), so we must use alternative emission lines as the reference for \(\nu_{\text{sys}}\) (as was done for many ordinary GNIRS-DQS sources; see Paper I). In spite of the larger intrinsic uncertainties associated with using the Mg II and Hβ emission lines as \(\nu_{\text{sys}}\) indicators (\(\sim 200\) km s\(^{-1}\) and \(\sim 400\) km s\(^{-1}\), respectively; Shen et al. 2016), these uncertainties are typically much smaller than the Blueshift([C IV]) values observed in the majority of luminous high-redshift quasars (see Paper I). Therefore, the lack of [O III]-based \(\nu_{\text{sys}}\) values for such sources should not affect the conclusions of this work significantly.

2.4. \(M_{\text{BH}}\) and \(L_{\text{Edd}}\) Estimates

Traditional estimation of single-epoch \(M_{\text{BH}}\) values has made use of the reverberation-mapping (RM) scaling relationship between the size of the Hβ-emitting region \(R_{\text{Hβ}}\) and \(\nu L_\beta(5100\) Å\) (e.g., Laor 1998; Wandel et al. 1999; Kaspi et al. 2005; Bentz et al. 2013). In this work, we use the empirical scaling relation established by Bentz et al. (2013) for consistency with other recent studies (e.g., Maithil et al. 2022; Paper II):

\[
\log \left[ \frac{R_{\text{Hβ}}}{\text{lt - days}} \right] = (1.527 \pm 0.031) \\
+ (0.533 \pm 0.035) \log \ell_{\text{44}},
\]

\[\ell_{\text{44}}\]
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<th>Quasar</th>
<th>$z_{\text{sys}}$</th>
<th>$\log \nu L_\nu(5100 \text{ Å})$ (erg s$^{-1}$)</th>
<th>FWHM(H$\beta$) (km s$^{-1}$)</th>
<th>$R_{\text{Fe II}}$</th>
<th>$\log M_{\text{BH}}$ ($M_\odot$)</th>
<th>$\log M_{\text{BH,corr}}$ ($M_\odot$)</th>
<th>$L/L_{\text{Edd}}$</th>
<th>$L/L_{\text{Edd,corr}}$</th>
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<th>Blueshift(C IV) (km s$^{-1}$)</th>
<th>$L/\nu L_\nu$</th>
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<td>7.36</td>
<td>0.33</td>
<td>0.41</td>
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</table>

Notes:

Only the first ten lines are shown.

- Source of rest-frame optical–UV data, Column (12): $z_{\text{sys}}$, $\nu L_\nu(5100 \text{ Å})$, FWHM(H$\beta$), $R_{\text{Fe II}}$; Column (13): EW(C IV), and Blueshift(C IV). (1) Paper I, (2) Paper II, (3) Boroson & Green (1992), (4) Baskin & Laor (2005).
- There are no errors reported for EW(C IV) and Blueshift(C IV) values for PG quasars in Baskin & Laor (2005).

(This table is available in its entirety in machine-readable form.)
where $\ell_{44} \equiv \nu L_{\nu}(5100 \ \text{Å}) / 10^{44} \ \text{erg s}^{-1}$.

However, the H/β RM sample was subsequently determined to be biased toward objects with strong, narrow [O III] emission lines and, in effect, is biased in favor of low-accretion-rate broad-line AGNs (see, e.g., Robinson 1994; Shen & Ho 2014). Recent RM campaigns aimed at minimizing such bias, such as the super-Eddington accreting massive black hole (SEAMBH, Du et al. 2014 Du et al. 2014, 2016, 2018) and the SDSS-RM project (Shen et al. 2015), found deviations from the traditional size–luminosity relationship. In particular, SEAMBHs found a population of rapidly accreting AGNs with a BEL size up to 3–8 times smaller than predicted by Equation (2), which implies an overestimation of super-Eddington-accreting $M_{\text{BH}}$ values from single-epoch spectra by the same factor. We apply a $R_{\text{Fe H}}$ correction to the traditional H/β-based $M_{\text{BH}}$ estimation, a method developed by Du & Wang (2019). The $R_{\text{Fe H}}$ parameter has been shown to correlate with $L/L_{\text{Edd}}$ (e.g., Netzer & Trakhtenbrot 2007).

For the Fe II-corrected values of $M_{\text{BH}}$ (hereafter, $M_{\text{BH,corr}}$), we apply the size–luminosity scaling relation for $R_{\text{H}}$, following Equation (5) of Du & Wang (2019):

$$
\begin{align*}
\log \left[ \frac{R_{\text{H}}(\text{corr})}{\text{lt days}} \right] = & (1.65 \pm 0.06) + (0.45 \pm 0.03) \log \ell_{44} \\
&+ (0.35 \pm 0.08) R_{\text{Fe H}}.
\end{align*}
$$

Subsequently, $M_{\text{BH}} (M_{\text{BH,corr}})$ can be estimated using the following relationship:

$$
M_{\text{BH}} (M_{\text{BH,corr}}) = f \left[ \frac{R_{\text{H}}(\text{corr})}{\text{pc}} \right] \left( \frac{\Delta V}{\text{km s}^{-1}} \right)^2 \left( \frac{G}{\text{pc M}_\odot^{-1} (\text{km s}^{-1})^2} \right)^{-1} \\
\approx 1.5 \left[ \frac{R_{\text{H}}(\text{corr})}{\text{pc}} \right] \left( \frac{\text{FWHM}(H/\beta)}{\text{km s}^{-1}} \right)^2 \\
\times \left( \frac{4.3 \times 10^{-3}}{\text{pc M}_\odot^{-1} (\text{km s}^{-1})^2} \right)^{-1} ,
$$

where we adopt $f = 1.5$ for the virial coefficient, consistent with results from Ho & Kim (2014), Yu et al. (2019, 2020a), and Maithil et al. (2022); $R_{\text{BELR}} \approx R_{\text{H}} (R_{\text{H}}(\text{corr}))$ is the size–luminosity relation from Equations (2) or (3); $\Delta V$ is the velocity width of the emission line, which is taken here as FWHM(H/β), assuming Doppler broadening (Wandel et al. 1999); and $G$ is the gravitational constant.

The $L/L_{\text{Edd}}$ parameter can be computed from the corresponding $M_{\text{BH}}$ value following Equation (2) of Shemmer et al. (2010) assuming that $L_{\text{Edd}}$ is computed for the case of solar metallicity:

$$
L/L_{\text{Edd}} (L/L_{\text{Edd, corr}}) = 1.06 f(L) \left( \frac{\nu L_{\nu}(5100 \ \text{Å})}{10^{44} \ \text{erg s}^{-1}} \right) \left( \frac{M_{\text{BH}} (M_{\text{BH, corr}})}{10^9 \text{M}_\odot} \right)^{-1} ,
$$

where $f(L)$ is the luminosity-dependent bolometric correction to $\nu L_{\nu}(5100 \ \text{Å})$, derived from Equation (21) of Marconi et al. (2004).

We note that a wide range of bolometric corrections for quasars is available in the literature (e.g., Richards et al. 2006; Nemmen & Brotherton 2010; Runnoe et al. 2012; Netzer 2019). However, in general, the range of these corrections is not large enough to affect the conclusion of our work. For example, Maithil et al. (2022) recently used a constant bolometric correction of $L_{\text{bol}}/\nu L_{\nu}(5100 \ \text{Å}) \sim 9$; the bolometric corrections we derive are in the range of $\sim 5–6$, which results in a relatively small systematic offset in the derivation of the $L/L_{\text{Edd}}$ parameter.

The uncertainties associated with the corrected $M_{\text{BH}}$ and $L/L_{\text{Edd}}$ values in this work are estimated to be at least $\sim 0.3$ dex (see Table 2 of Maithil et al. 2022) but could be much larger ($\sim 0.4–0.6$ dex) for high $L/L_{\text{Edd}}$ objects such as WLQs (see also SL15).

### 3. Results and Discussion

#### 3.1. Black Hole Masses and Accretion Rates

For the 248 quasars included in this work, we determine the virial H/β-based $M_{\text{BH,corr}}$ and corresponding $L/L_{\text{Edd,corr}}$ values from their derived optical properties, following the Fe II-corrected BELR size–luminosity relation of Equation (3), applied to Equations (4) and (5). We also calculate these quasars’ $M_{\text{BH}}$ and $L/L_{\text{Edd}}$ values following the traditional BELR size–luminosity relation of Equation (2) to compare the two methods for estimating black hole masses and accretion rates.

Figure 1 presents the traditional versus corrected $M_{\text{BH}}$ and $L/L_{\text{Edd}}$ values for the quasars in our sample, following the procedure of Maithil et al. (2022). The H/β-based $M_{\text{BH,corr}}$ values of ordinary quasars show small systematic deviations from the traditional BELR size–luminosity relation estimates (less than a factor of 2 for 222 out of 230 quasars). On the other hand, for a majority of the WLQs, due to the relative weakness in H/β emission compared to the Fe II emission, $M_{\text{BH,corr}}$ values deviate significantly from the traditional relation, up to 1 order of magnitude. Since $L/L_{\text{Edd}}$ is inversely proportional to $M_{\text{BH}}$, the $L/L_{\text{Edd,corr}}$ values are enhanced by a similar factor. This result is in line with the Maithil et al. (2022) finding of a larger deviation from the one-to-one relation in high-accretion-rate quasars.

#### 3.2. The Anticorrelation between EW(C IV) and LA_{Edd}

We use our sample to explore the anticorrelation between EW(C IV) and H/β-based $L/L_{\text{Edd}}$, previously studied in SL15 (i.e., the MBE), as well as with $L/L_{\text{Edd,corr}}$. Figure 2 shows EW(C IV) plotted against the traditional $L/L_{\text{Edd}}$ values (left) and against the Fe II-corrected $L/L_{\text{Edd,corr}}$ values (right). The first four rows of Table 3 present the respective Spearman rank correlation coefficients ($r_s$) and chance probabilities ($p$) of the ordinary quasar sample and the complete sample, including WLQs, for the correlation involving EW(C IV). We detect significant anticorrelations between EW(C IV) and $L/L_{\text{Edd}}$ both with and without WLQs (i.e., $p \ll 1\%$). However, the anticorrelation for the sample including WLQs is slightly weaker than that without WLQs (both $p$ values are roughly similar, but $r_s$ increases slightly). Our result reaffirms findings by SL15, who found WLQs to be outliers in this relation.

With an Fe II correction, the $L/L_{\text{Edd,corr}}$ values provide a significantly stronger anticorrelation with EW(C IV) as the $r_s$ value decreases from $-0.36$ (for the $L/L_{\text{Edd}}$ case) to $-0.48$. Furthermore, the inclusion of WLQs no longer spoils the Spearman rank correlation; in fact, the $p$ value remains extremely low ($p = 4.02 \times 10^{-20}$ for the entire sample), and the $r_s$ value decreases from $-0.48$ to $-0.54$, indicative of a stronger anticorrelation. Nevertheless, the $L/L_{\text{Edd,corr}}$ values of most of the WLQs in our sample still appear considerably smaller than a
linear model would suggest (see Figure 2). To quantify the deviation of WLQs from the MBE, we fit a simple linear model, without considering the errors, to the log EW(C IV) and log \( L/L_{\text{Edd,corr}} \) values of the ordinary quasar sample. Our WLQs deviate from the best-fit model by a mean of \(-3.4\sigma\), with a range in deviation from 1.08\( \sigma \) to 8.02\( \sigma \). Such a discrepancy paints WLQs as significant outliers in this correlation.

We also explore whether a bolometric luminosity correction based on the peculiarity of WLQs could account for this discrepancy. Although several methods for correcting bolometric luminosity are available in the literature (e.g., Richards et al. 2006; Nemmen & Brotherton 2010; Runnoe et al. 2012; Netzer 2019), if the Eddington ratios of WLQs were to be reliably predicted by the MBE, these corrections must be up to \( \sim 10^5 \) times larger than those of Marconi et al. (2004); as in the case of SDSS J1141+0219 with EW(C IV) = 0.4 Å. Such a discrepancy is larger than the difference expected by any of the current bolometric correction methods in the literature. These results reveal that EW(C IV) is likely not the sole indicator of quasars’ accretion rates.

3.3. The C IV || Distance as an Indicator of L/L_{Edd}

Rivera et al. (2020) used an independent component analysis technique to analyze the spectral properties of the C IV emission line in 133 quasars from the SDSS-RM project (Shen et al. 2015). In particular, they fitted a piecewise polynomial to trace the positions of these sources on the EW(C IV) and the Blueshift...
The projected position of a quasar along this curve is defined as its \( C IV || Distance \). To calculate the value of this \( C IV || Distance \) parameter, we follow the procedure summarized in R22 and detailed in McCaffrey & Richards (2021). In short, we first transform the values of the two axes (EW(\( C IV \)) and Blueshift (\( C IV \))) to lie between 0 and 1, using the MinMaxScaler function within \texttt{scikit-learn} (Pedregosa et al. 2011). Then, the \( C IV || Distance \) values are measured relative to the first point of the best-fit curve, located at EW(\( C IV \)) = 316 Å and Blueshift (\( C IV \)) ≈ 50 km s\(^{-1}\). This parameter essentially indicates the location along a nonlinear first principal component of the \( C IV \) parameter space and encodes information about the physics of the \( C IV \)-emitting gas (e.g., Richards et al. 2011, 2021; Giustini & Proga 2019).

The left panel of Figure 3 shows the distribution of EW(\( C IV \)) versus Blueshift(\( C IV \)) of the 248 quasars in our sample. One quasar from BL04, PG 1202+281, is not shown in the left panel due to its extremely high EW(\( C IV \)) = 290 Å. Right panel: illustration of the \( C IV || Distance \) parameter. The data are first scaled so that the two axes share the same limit; then, each data point is projected onto the best-fit curve obtained from R22. The \( C IV || Distance \) value of each quasar is defined as its projected position (green point) along the solid black curve. Three of the WLQs, SDSS J114153.33+021924.4, SDSS J123743.07+630144.7, and SDSS J094602.31+274407.0, and one ordinary quasar, PG 1202+281, are not shown in the right panel for clarity, but only their projected positions onto the curve are relevant to our results.

Figure 4. \( C IV || Distance \) vs. \( L/L_{\text{Edd}} \) of 248 quasars in our sample. In the left panel, the \( C IV || Distance \) values are plotted against the traditional H\( \beta \)-based \( L/L_{\text{Edd}} \) parameter and in the right panel, against the Fe II-corrected H\( \beta \)-based \( L/L_{\text{Edd,corr}} \) parameter. The ordinary quasar PG 1202+281 with \( L/L_{\text{Edd,corr}} \) = 0.06 and \( C IV || Distance \) = 0.02 is not plotted for clarity. The correlation for the ordinary quasar sample, obtained by fitting a linear model, is shown as a dashed line. The shaded regions represent the 1\( \sigma \) and 2\( \sigma \) deviations from the fitted correlation. While using the traditional size–luminosity relation to estimate accretion rates already yields a strong correlation, the Fe II-corrected accretion rates show a much stronger correlation with the \( C IV || Distance \) parameter for all quasars. Furthermore, this parameter serves as a better predictor for \( L/L_{\text{Edd,corr}} \) than for \( L/L_{\text{Edd}} \).
samples that are different from those of R22, the best-fit curve traces the CIV parameter space of sources across wide ranges of redshifts and luminosities. Since all quasars in our sample are selected photometrically, either in optical (for GNIRS-DQS quasars) or UV (for BL04 quasars) surveys, and were not selected based on their spectroscopic characteristics, there are no known biases associated with their selection, and thus they are expected to trace the CIV parameter space in a similar manner to larger samples of quasars in other studies (e.g., see also Rankine et al. 2020).

While the EW(C IV) parameter, on its own, is not an ideal accretion-rate indicator, the CIV ||Distance parameter appears to provide a robust indication of the accretion rates for all quasars including WLQs. We plot the C IV ||Distance versus Hβ-based L/L_{Edd} (left) and L/L_{Edd,corr} (right) for all quasars in our sample in Figure 4. The last four rows of Table 3 provide the Spearman rank correlation coefficients and chance probabilities for the correlations involving the C IV ||Distance. Both the L/L_{Edd} and L/L_{Edd,corr} are significantly correlated with the C IV ||Distance parameter (i.e., p ≤ 1%).

In the case of C IV ||Distance versus L/L_{Edd,corr}, the correlation coefficient is considerably larger than the correlation involving L/L_{Edd} (0.57 versus 0.36), indicating the importance of the Fe II correction to M_{BH}. Furthermore, the inclusion of WLQs in the sample both strengthens the correlation (r_s increases from 0.52 to 0.57, while the p value remains extremely small, <10^{-16}) and allows the high-L/L_{Edd,corr} end of the correlation to be more populated. There is also no significant deviation of the WLQs from this correlation, as opposed to their behavior in the MBE (see Figure 2), as well as in the C IV ||Distance versus traditional L/L_{Edd} (see left panel of Figure 4). To quantify this effect, we fit a linear model to the C IV ||Distance and L/L_{Edd} (L/L_{Edd,corr}) space, taking into account only the ordinary quasars. Then, we calculate the mean scatter of the WLQs from this line. In the case of L/L_{Edd}, we find the deviation from the best-fit line to range from 0.62σ to 2.96σ, and the mean deviation to be ∼1.8σ. Meanwhile, the deviation in the case of L/L_{Edd,corr} ranges from 0.01σ to 2.18σ, with a mean deviation of ∼1.1σ. Thus, using L/L_{Edd,corr} not only results in a stronger correlation with C IV ||Distance, but C IV ||Distance also serves as a better predictor for L/L_{Edd,corr} than for L/L_{Edd}.

The right panel of Figure 4 shows that WLQs are not a disjoint subset of quasars in the UV-optical space (see also Martínez-Aldama et al. 2018). Our results indicate that WLQs possess relatively high accretion rates due not only to their extremely weak C IV lines but rather to their relatively large values of the C IV ||Distance parameter. Similarly, we observe quasars with high accretion rates (and large values of C IV ||Distance) that do not necessarily possess extremely weak C IV lines, some of which have Eddington ratios that are larger than those of several WLQs. Finally, while we are unaware of a large population of quasars that deviate significantly from the correlations of Figure 4, a future examination of, e.g., Hβ-based L/L_{Edd} values of quasars with very large EW(C IV) (e.g., Fu et al. 2022) is warranted to further test our results.

In this work, we show that the C IV and Hβ parameter space provides important diagnostics for quasar physics. In particular, we found that the C IV ||Distance can serve as a robust predictor of a quasar’s accretion rate, especially after a correction based on R_{Fe II} is applied. Within the limits of our sample, we also find that WLQs are not a disjoint subset of the type 1 quasar population but instead lie preferentially toward the extreme end of the C IV–Hβ parameter space.

4. Conclusions

We compile a statistically meaningful sample of ordinary quasars and WLQs to study the dependence of quasar accretion rates, corrected for the relative strength of Fe II emission with respect to Hβ, upon source location in the C IV parameter space. Utilizing 18 WLQs, 16 of which are obtained from the literature and two of which are presented in this work, we confirm the findings of Mathil et al. (2022) that the traditional approach to estimating the Eddington ratio for rapidly accreting quasars systematically underestimates this property by up to an order of magnitude compared to Fe II-corrected values of this parameter.

Using the Fe II-corrected values of Hβ-based L/L_{Edd} we investigate the correlation between this parameter and the C IV parameter space. We confirm and strengthen the SL15 results by finding that WLQs spoil the anticorrelation between EW(C IV) and Hβ-based L/L_{Edd} for quasars, whether the latter parameter is estimated using the traditional method or whether a correction based on Fe II emission is employed in the M_{BH} estimate. In keeping with SL15, we conclude that the EW(C IV) cannot be the sole indicator of accretion rate in quasars.

We also investigate the relationships between a recently introduced parameter, the C IV ||Distance, which is a combination of EW(C IV) and Blueshift(C IV), and the traditional Hβ-based L/L_{Edd} and the Fe II-corrected L/L_{Edd,corr}. Such relationships yield strong correlations, especially in the case of Fe II-corrected L/L_{Edd,corr}, and can accommodate all the quasars in our sample. Our finding suggests that WLQs are not a disjoint subset of sources from the general population of quasars. We find that many WLQs have extremely high accretion rates, which is indicated by their preferentially higher values of the C IV ||Distance parameter. Similarly, we find several quasars in our sample that possess high Eddington ratios and correspondingly large values of the C IV ||Distance, which do not have extremely weak C IV lines; some of these sources display Eddington ratios that are larger than those of a subset of our WLQs.

In the context of the C IV parameter space, it will be interesting to investigate whether the extreme X-ray properties of WLQs are the result of extremely large C IV ||Distance values rather than resulting only from extremely weak C IV lines. Such a test would require X-ray coverage of a large sample of sources with
Hβ + Fe II data across the widest possible C IV parameter space such as the GNIRS-DQS sample of Paper I. It would also be useful to determine whether the weakness of the broad Lyα + N V emission line complex (from which the first high-redshift WLQs were identified) also correlates with C IV || Distance, which will require rest-frame ultraviolet spectroscopy (Paul et al. 2022). The results of these investigations will shed new light on the connection between the quasar accretion rate and the physics of the inner accretion disk and BELR.

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Appendix

NIR Spectroscopy of SDSS J1137+3919 and SDSS J2137−0039

SDSS J113747.64+391941.5 and SDSS J213742.25−003912.7 (hereafter, SDSS J1137+3919 and SDSS J2137−0039, respectively) are two WLQs with redshifts suitable for observing the Hβ line in the H Band. Observations of these quasars were carried out by the Gemini-North Observatory using GNIRS throughout four observing runs between 2014 March 14 and 2014 August 4, under program GN-2014A-Q-47. The observation log appears in Table A1. For both targets, we used the short blue camera, with spatial resolution 0.15 pix−1 and a 1′ 0 slit to achieve a spectral resolution of R ∼ 600. An H filter was applied, producing a spectral range of 1.5–1.8 μm, corresponding to rest-frame ∼4500–5300 Å. Exposure times for each subintegration were 238 and 220 s, and the total integration times were 7140 and 7040 s for SDSS J1137+3919 and SDSS J2137−0039, respectively. These observations were performed using the standard “ABBA” nodding pattern of the targets along the slit in order to obtain primary background subtraction.

The spectra were processed using the standard procedure of the IRAF Gemini package based on the PyRAF Python-based interface. Exposures from the same nodding position were added to boost the signal-to-noise ratio; then, the sum of exposures from two different nodding positions were subtracted to remove background noise. Wavelength calibration was done against an argon lamp in order to assign wavelength values to the observed pixels.

Spectra of telluric standard stars with T eff ∼ 9700 K were taken immediately before or after the science exposures to remove telluric absorption features in the quasars’ observed spectra. These spectra were processed in a similar fashion, followed by a removal of the stars’ intrinsic hydrogen absorption lines by fitting a Lorentzian profile to each hydrogen absorption line and interpolating across this feature to connect the continuum on each side of the line. The quasars’ spectra were divided by the corrected stellar spectra. The corrected quasar spectra were then multiplied by an artificial blackbody curve with a temperature corresponding to the telluric standard star, which yielded a cleaned, observed-frame quasar spectrum.

Flux calibrations were obtained by taking the Wide-field Infrared Survey Explorer (Wright et al. 2010) W1-band (at 3.4 μm) apparent magnitudes, reported by SDSS Data Release 16 (Lyke et al. 2020), and the W1 isophotal flux density Fν(iso) given in Table 1 of Jarrett et al. (2011). Flux densities at 3.4 μm were derived according to

\[ F_\nu(3.4 \mu m) = F_\nu(iso) \cdot 10^{-mag/2.5}. \]  

The flux densities at 3.4 μm were extrapolated to flux densities at 1.63 μm, roughly corresponding to λrest = 5100 Å, assuming an optical continuum of the form \( F_\nu \propto \nu^{-0.5} \) (e.g., Vanden Berk et al. 2001).

We modeled the spectra following the methods of Shemmer et al. (2004) and Shemmer et al. (2010). Our model consists of a linear continuum through the average flux densities of two narrow (∼20 Å) rest-frame bands centered on 4750 and 4975 Å, a broadened Fe II emission template (Boroson & Green 1992), and two Gaussian profiles for the Hβ λ4861 emission line. No [O III] emission lines are detectable in either spectrum, and we placed upper limits on their EWs by fitting a Gaussian feature where the [O III] emission lines should be such that they are indistinguishable from the continuum. The final, calibrated near-infrared spectra of the two WLQs appear in Figure A1.

**Table A1**

<table>
<thead>
<tr>
<th>Quasar (SDSS J)</th>
<th>z</th>
<th>zsys</th>
<th>log L5100 Å (erg s⁻¹)</th>
<th>Observation Dates</th>
<th>Exp. Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>113747.64+391941.5</td>
<td>2.420</td>
<td>2.428</td>
<td>45.8</td>
<td>2014 Mar 14, 20</td>
<td>7140</td>
</tr>
<tr>
<td>213742.25−003912.7</td>
<td>2.281</td>
<td>2.294</td>
<td>45.8</td>
<td>2014 Jun 29, Aug 04</td>
<td>7040</td>
</tr>
</tbody>
</table>

**Notes.**

a Obtained from visually inspected redshifts (zvis) reported in SDSS Data Release 16 (Lyke et al. 2020).

b Systemic redshifts (see the Appendix for details).
In both sources we detected weak and relatively narrow Hβ lines as well as strong Fe II features compared to quasars at similar luminosities and redshifts (e.g., see Netzer et al. 2007; Shen 2016). We also determined the systemic redshift ($z_{sys}$) values from the observed-frame wavelength of the peak ($\lambda_{peak}$) of the Hβ emission line, a similar treatment as in Matthews et al. (2021) for sources that lack [O III] emission. The $z_{sys}$ values are larger than the redshifts reported by Lyke et al. (2020) by $\Delta z = 0.008$ in SDSS J1137+3919 and by $\Delta z = 0.013$ in SDSS J2137−0039, corresponding to velocity offsets (blueshifts) of 700 km s$^{-1}$ and 1184 km s$^{-1}$, respectively, which is consistent with typical velocity offsets between SDSS Pipeline redshifts and $z_{sys}$ values observed in luminous, high-redshift quasars (Dix et al. 2020; Paper I). The rest-frame spectra in Figure A1 have henceforth been corrected by $z_{sys}$. Rest-frame EWs of Hβ $\lambda 4861$, Fe II $\lambda 4434–4684$, and the upper limit on the EWs of [O III] $\lambda 5007$ were calculated for SDSS J1137+3919 to be 16 Å, 53 Å, and ≤4 Å, and for SDSS J2137−0039 to be 20 Å, 49 Å, and ≤5 Å, respectively. The flux densities at a rest-frame wavelength of 5100 Å are $7.77 \times 10^{-18}$ erg cm$^{-2}$ s$^{-1}$ Å$^{-1}$ and $8.18 \times 10^{-18}$ erg cm$^{-2}$ s$^{-1}$ Å$^{-1}$, respectively.

**Figure A1.** The NIR spectra of SDSS J1137+3919 (top) and SDSS J2137−0039 (bottom). In each panel, the continuous line is the observed spectrum of each quasar. The continuous straight line below the spectrum is the linear continuum fit. The dashed line is the Hβ $\lambda 4861$ profile modeled with two Gaussians. The dotted--dashed line is the Fe II template from Boroson & Green (1992), which was broadened by 1500 km s$^{-1}$ for SDSS J1137+3919 and 1400 km s$^{-1}$ for J2137−0039. The bold solid line is the entire fitted spectrum.

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